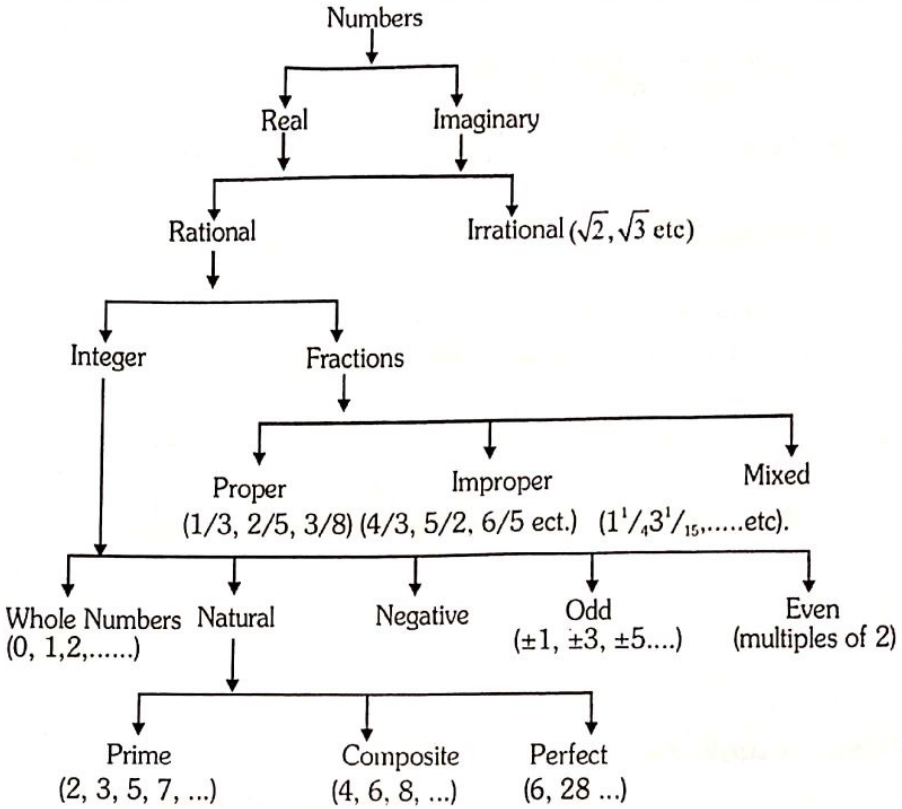
CHAPTER 1

Number System

1.1 System of numbers



Natural numbers

Counting numbers 1, 2, 3, 4, 5,.....are known as natural numbers. The set of all natural numbers can be represented by

$$N = \{1, 2, 3, 4, 5....\}$$

Whole numbers

If we include 0 among the natural numbers, then the numbers 0, 1, 2, 3, 4, 5.....are called whole numbers. The set of whole numbers can be represented by

$$W = \{0, 1, 2, 3, 4, 5,.... \}$$

Clearly, every natural number is a whole number but 0 is a whole number which is not a natural number.

Integers

All counting numbers and their negatives including zero are known as integers. The set of integers can be represented by

$$Z \text{ or } \{.....-4, -3, -2, -1, 0, 1, 2, 3, 4,.....\}$$

Positive integers

The set $I^+ = \{1, 2, 3, 4,....\}$ is the set of all positive integers. Clearly, positive integers and natural numbers are synonyms.

Negative integers

The set $I^- = \{-1, -2, -3,.....\}$ is the set of all negative integers. 0 is neither positive nor negative.

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Non-negative integers

The set {0, 1, 2, 3,.....} is the set of all non-negative integers.

Rational numbers

The numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, are known as rational numbers, e.g. $\frac{4}{7}, \frac{3}{2}, \frac{5}{8}, \frac{0}{1}, -\frac{2}{3}$, etc. The set of all rational numbers is denoted by Q.

i.e. $Q = \left\{x : x = \frac{p}{q}; p, q \neq I, q \neq 0\right\}$

Since every natural number a can be written as $\frac{a}{1}$, so is a rational number. Since 0 can be written as $\frac{0}{1}$ and every non-zero integer 'a' can be written as $\frac{a}{1}$, so it is also a rational number.

Every rational number has a peculiar characteristic that when expressed in decimal form is expressible either in terminating decimals or non-terminating repeating decimals.

For example, $\frac{1}{5} = 0.2, \frac{1}{3} = 0.333..., \frac{22}{7} = 3.1428714287, \frac{8}{44} = 0.18188 ...., \text{etc.}$

The recurring decimals have been given a short notation as

$0.333... = 0.\overline{3}$

$4.1555... = 4.0\overline{5}$

$0.323232... = 0.\overline{32}$

Irrational numbers

Those numbers which when expressed in decimal form are neither terminating nor repeating decimals are known as irrational numbers, e.g. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \pi$ etc.

- Note, that the exact value of π is not $\frac{22}{7} \cdot \frac{22}{7}$ is rational while π is irrational number. $\frac{22}{7}$ is approximate value of π. Similarly, 3.14 is not an exact value of it.

Real numbers

The rational and irrational numbers combined together are called real numbers, e.g. $\frac{13}{21}, \frac{2}{5}, -\frac{3}{7}, \sqrt{3}, 4+\sqrt{2}$, etc. are real numbers. The set of real numbers is denoted by R.

- Note, that the sum, difference or product of a rational and irrational number is irrational.

  e.g. $3+\sqrt{2}, 4-\sqrt{3}, \frac{2}{3}-\sqrt{5}, 4\sqrt{3}, -7\sqrt{5}$ are all irrational.

Even numbers

All those numbers which are exactly divisible by 2 are called even numbers, e.g. 2,6,8,10 etc. are even numbers.

Odd numbers

All those numbers which are not exactly divisible by 2 are called odd numbers, e.g. 1,3,5,7, etc, are odd numbers.

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Prime numbers

Except 1 each natural number which is divisible by only 1 and itself is called as prime number e.g., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31,.....etc.

- There are total 25 prime numbers upto 100.
- There are total 46 prime numbers upto 200.
- 2 is the only even prime number and the least prime number.
- 1 is neither prime nor composite number.
- There are infinite prime numbers.
- A list of all prime numbers upto 100 is given below.

Table of Prime Numbers (1-100)

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Test to find whether a given number is a prime

Step 1 Select a least positive integer n such that n^2 > given number.

Step 2 Test the divisibility of given number by every prime number less than n.

Step 3 The given number is prime only if it is not divisible by any of these primes.

NUMERICAL CHALLENGE 1.1

(7) Investigate whether 571 is a prime number.

Solution

Since $(23)^2 = 529 < 571$ and $(24)^2 = 576 > 571$

$\therefore$ n=24

Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. Since 24 is divisible by 2, 571 is not a prime number.

Co-prime

A pair of two natural numbers having no common factor, other than 1, is called a pair of co-primes.

For example: (3, 5), (4, 5), (5, 6), (7, 9), (6, 7) etc., are co-primes.

Twin primes

Prime numbers differing by 2 are called twin primes, e.g. (3, 5), (5, 7), (11, 13) etc, are called twin primes

Prime triplet

A set of three consecutive primes differing by 2, such as (3, 5, 7) is called a prime triplet

"every prime number except 2 is odd but every odd number need not be prime."

Fractions

(a) Common fraction : Fractions whose denominator is not 10.

(b) Decimal fraction : Fractions whose denominator is 10 or any power of 10.

(c) Proper fraction : Numerator < Denominator i.e. $\frac{3}{5}$.

(d) Improper fraction : Numerator > Denominator i.e. $\frac{5}{3}$.

(e) Mixed fraction : Consists of integral as well as fractional part i.e. $3\frac{2}{7}$.

(f) Compound fraction : Fraction whose numerator and denominator themselves are fractions i.e. $\frac{2/}{5/}$

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Composite numbers

All natural numbers, which are not prime are composite numbers. If C is the set of composite number than C = {4, 6, 8, 9, 10, 12,....}

Imaginary numbers

All the numbers whose square is negative are called imaginary numbers. e.g. 3i, –4i,....; where $i = \sqrt{-1}$.

Complex numbers

The combined form of real and imaginary numbers is known as complex number.

It is denote by Z = A + iB, where A is real and B is imaginary part of Z and A, B $\in$ R.

1.2 Square & Square roots

The second power of number is called the square of that number. In other words the square of a number is the product of the number with the number itself.

- A given number is a perfect square, if it is expressed as a product of pairs of equal factors.

Important properties

(i) A natural number having 2, 3, 7 or 8 in the unit's place is never a perfect square (or squared number.)
17, 23, 118, 222 are not perfect squares.

(ii) The square of an even number is always an even number.
2^2=4, 6^2=36, 10^2=100, 12^2=144.

(iii) The square of an odd number is always an odd number.
3^2=9, 7^2=49, 13^2=169, 15^2=225.

(iv) The number of zeroes at the end of a perfect square is never odd.
100, 400, 3600, 640000 are perfect squares and 1000, 4000, 6400000 are not perfect squares.

(v) The square of a natural number n is equal to the sum of the first n odd numbers.
1^2=1= sum of the first 1 odd number.
2^2=1+3 = sum of the first 2 odd numbers.
3^2=1+3+5= sum of the first 3 odd numbers.

(vi) For every natural number n,
$(n+1)^2-n^2=(n+1+n)(n+1-n)=(n+1)+n$
$4^2-3^2=(3+1)+3=7$.
$16^2-15^2=(15+1)+15=31$.

(vii) A perfect square (other than 1) is either a multiple of 3 or exceeds a multiple of 3 by 1.
$49=(7)^2=3\times16+1$, $169=(13)^2=3\times56+1$.

(viii) A perfect square (other than 1) is either a multiple of 4 or exceeds a multiple of 4 by 4.
$441=(21)^2=4\times110+1$.

Some other properties

(i) If the unit digit of the number is zero then the unit digit of the square of this number will also be zero and the number of zeroes will be double in the square than that of its root.
e.g., $(60)^2$ = 3600, $(130)^2$ = 16900

(ii) If the unit digit of the number is 5 then the unit digit of its square is also 5 and the number formed by last two digits is 25.
e.g. $(35)^2$ = 1225, $(45)^2$ = 2025, $(55)^2$ = 3025 etc.

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(iii) If the unit digit of any number is 1 or 9 then the unit digit of the square of its number is always 1.
e.g., $(71)^2$ = 5041, $(31)^2$ = 961, $(19)^2$ = 361

(iv) If the unit digit of any number is 2 or 8 then the unit digit of the square of its number is always 4.

(v) If the unit digit of any number is 3 or 7 then the unit digit of its square is always 9.
e.g., $(23)^2$ = 529, $(27)^2$ = 729

(vi) If the unit digit of any number is 4 or 6 then the unit digit of its square is always 6.
e.g., $(26)^2$ = 676, $(24)^2$ = 576,
$(14)^2$ = 196, $(16)^2$ = 256 etc.

(vii) The square of any number is always positive irrespective of the nature of the given number.

(viii) Non square numbers between the squares of two consecutive natural numbers n & n +1 $\rightarrow (n+1)^2-n^2-1$
$= n^2+1+2n-n^2-1 = 2n$

(ix) If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

(x) Square root of a negative integer is an imaginary number.

(xi) Square roots of integers that are not perfect squares are always irrational numbers.

(xii) Every composite number can be uniquely factored as a product of prime numbers only.

(xiii) If a perfect square is of n-digits, then its square root will have n/2 digits if n is even or $\frac{(n+1)}{2}$ if n is odd.

1.3 Cube & cube roots

Cube of a number is obtained by multiplying the number by itself thrice.

For example, 27 is the cube of 3 as 27 = 3 ×3 ×3.

Cube root

Cube root of a given number is that number which when raised to the third power produces the given number, that is the cube root of a number x is the number whose cube is x.

- The cube root of x is written as $\sqrt[3]{x}$.

  For example, cube root of 64 is 4 as 4 ×4 ×4 = 64.

Short-cut method of finding cube roots of exact cubes consisting up to 6 digits:

Before we discuss the method to find the cube roots of exact cubes, the following two remarks are very useful and must be remembered by heart.

(i) 1^3= 1; 2^3= 8; 3^3 = 27; 4^3= 64; 5^3= 125; 6^3=216; 7^3= 343; 8^3= 512; 9^3 = 729; 10^3=1000.

(ii) If the cube ends in 1, then its cube root ends in 1
If the cube ends in 2, then its cube root ends in 8
If the cube ends in 3, then its cube root ends in 7
If the cube ends in 4, then its cube root ends in 4
If the cube ends in 5, then its cube root ends in 5
If the cube ends in 6, then its cube root ends in 6
If the cube ends in 7, then its cube root ends in 3
If the cube ends in 8, then its cube root ends in 2
If the cube ends in 9, then its cube root ends in 9
If the cube ends in 0, then its cube root ends in 0

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Clearly from above

$1 \leftrightarrow 1, 4 \leftrightarrow 4, 5 \leftrightarrow 5, 6 \leftrightarrow 6, 9 \leftrightarrow 9, 0 \leftrightarrow 0$

$2 \leftrightarrow 8, 3 \leftrightarrow 7.$

Some other properties

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

(iii) The cube of a natural number which is a multiple of 3 is a multiple of 27.

(iv) The cube of a natural number which of the form $3n + 1$ (e.g., 4, 7,10.....) is also a number of the form $3n + 1$.

(v) The cube of natural number which is of the form $3n + 2$ (e.g., 5, 8, 11,.....) is also a number of the form $3n + 2$.

1.4 Squares (short-cut methods)

To square any number ending with 5.

Method : $(A5)^2 = A(A+1)/25$

To square a number in which every digit is one.

Method: Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.

To square number which is nearer to 10x.

Method: Use the formula.

$x^2 = (x^2-y^2) + y^2 = (x+y)(x-y)+y^2$

NUMERICAL CHALLENGE 1.2

Find the squares of following:

(i) $(25)^2 = 2(2+1)/25=6/25 = 625$

(ii) $(45^2) = 4(4+1)/25 = 20/25 = 2025$

(iii) $(85)^2 = 8(8+1)/25 = 72/25 = 7225$

(iv) $11^2 = 121$

(v) $11^2 = 12321$

(vi) $1111^2 = 1234321$

(vii) $222^2 = 2^2(111)^2 = 4(12321) = 49284$

(viii) $3333^2 = (1111)^2 = 9(1234321) = 11108889$

(ix) $(97)^2 = (97+3)(97-3)+3^2 = 9400+9=9409$

(x) $(102)^2 = (102-2)(102+2)+2^2 = 10400+4=10404$

(xi) $(994)^2 = (994+6)(994-6) + 6^2 = 988000 + 36 = 988036$

(xii) $(1005)^2 = (1005-5)(1005+5) + 5^2 = 1010000 + 25 = 1010025$

1.5 Multiplication (short-cut methods)

Multiplication of a given number by 9, 99, 999, etc., that is by 10^{n-1}

Method: Put as many zeroes to the right of the multiplication as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

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NUMERICAL CHALLENGE 1.3

Multiply

(i) 3893 by 99 (ii) 4327 by 999 (iii) 5863 by 9999

Solution

(i) $3893 \times 99 = 389300 - 3893 = 385407.$

(ii) $4327 \times 999 = 4327000 - 4327 = 4322673$

(iii) $5863 \times 9999 = 58630000 - 5863 = 58624137$

Multiplication of a given number by 11, 101, 1001, etc., that is by 10^n+1.

Method: Place n zeroes to the right of the multiplicant and then add the multiplicant to the number so obtained.

NUMERICAL CHALLENGE 1.4

Multiply

(i) 4782×11 (ii) 9836×101 (iii) 6538×1001

Solution

(i) $4782 \times 11 = 47820 + 4782 = 52602$

(ii) $9836 \times 101 = 983600 + 9836 = 993436$

(iii) $6538 \times 1001 = 6538000 + 6538 = 6544538$

Multiplication of a given number by 15, 25, 35, etc.

Method: Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

NUMERICAL CHALLENGE 1.5

Multiply

(i) 7054×15 (ii) 3897×25 (iii) 4563×35

Solution

(i) $7054 \times 15 = \frac{1}{2}(7054 \times 30) = \frac{1}{2}(211620) = 105810$

(ii) $3897 \times 25 = \frac{1}{2}(3897 \times 50) = \frac{1}{2}(194850) = 97425$

(iii) $4563 \times 35 = \frac{1}{2}(4563 \times 70) = \frac{1}{2}(319410) = 159705$

Multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.

Method: Place as many zeroes to the right of the multplicant as in the power of 5 in the multiplier, then divide the number so obtained by 2 raised to the same power as is the power of 5.

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NUMERICAL CHALLENGE 1.6

Multiply

(i) 3982×5 (ii) 4739×25 (iii) 7894×125 (iv) 4863×625

Solution:

(i) $3982 \times 5 = \frac{39820}{2} = 19910$

(ii) $4739 \times 25 = \frac{473900}{2^2} = \frac{473900}{4} = 118475$

(iii) $7894 \times 125 = \frac{7894000}{2^3} = \frac{7894000}{8} = 986750$

(iv) $4863 \times 625 = \frac{48630000}{2^4} = \frac{48630000}{16} = 3039375$

1.6 Test of divisibility

Divisibility by 2: A number is divisible by 2 if the unit's digit is zero or divisible by 2.

For example, 4, 12, 30, 18, 102, etc. are all divisible by 2.

Divisibility by 3: A number is divisible by 3 if the sum of digits in the number is divisible by 3.

For example, the number 3792 is divisible by 3 since $3 + 7 + 9 + 2 = 21$, which is divisible by 3.

Divisibility by 4: A number is divisible by 4 if the number formed by the last two digits (ten's digit and unit's digit) is divisible by 4 or are both zero.

For example, the number 2616 is divisible by 4 since 16 is divisible by 4.

Divisibility by 5: A number is divisible by 5 if the unit's digit in the number is 0 or 5.

For example, 13520, 7805, 640, 745, etc. are all divisible by 5.

Divisibility by 6: A number is divisible by 6 if the number is even and sum of its digits is divisible by 3.

For example, the number 4518 is divisible by 6 since it is even and sum of its digits $4+5+1+8=18$ is divisible by 3.

Divisibility by 7: The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the remainder is divisible by 7, then the given number is also divisible by 7.

For example, consider the number 448. On doubling the unit digit 8 of 448 we get 16.

Then, $44-16 = 28$.

Since 28 is divisible by 7, 448 is divisible by 7.

Divisibility by 8: A number is divisible by 8, if the number formed by the last 3 digits is divisible by 8.

For example, the number 41784 is divisible by 8 as the number formed by last three digits i.e. 784 is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of its digits is divisible by 9.

For example, the number 19044 is divisible by 9 as the sum of its digits $1 + 9 + 0 + 4 + 4 = 18$ is divisible by 9.

Divisibility by 10: A number is divisible by 10, if ends in zero.

For example, the last digit of 580 is zero, therefore 580 is divisible by 10.

Divisibility by 11: A number is divisible by 11, if the difference of the sum of the digits at odd places and sum of the digits at even places is either zero or divisible by 11.

For example, in the number 38797, the sum of the digits at odd places is $3 + 7 + 7 = 17$ and the sum of the digits at even places is $8 + 9 = 17$. The difference is $17 - 17 = 0$, so the number is divisible by 11.

Divisibility by 12: A number is divisible by 12 if it is divisible by 3 and 4.

Divisibility by 25: A number is divisible by 25 if the number formed by the last two digits is divisible by 25 or last two digits are zero.

For example, the number 13675 is divisible by 25 as the number formed by the last two digits is 75 which is divisible by 25.

Divisibility by 125: A number is divisible by 125 if the number formed by the last three digits is divisible by 125 or the last three digits are zero.

For example, the number 5250 is divisible by 125 as 250 is divisible by 125.

Divisibility by 18: An even number satisfying the divisibility test of 9 is divisible by 18.

Divisibility by 88: A number is divisible by 88 if it is divisible by 11 and 8.

1.7 Decimal fractions

Decimal fraction : Fractions in which the denominators are the powers of 10 are called decimal fractions.

In general, the decimal fractions are of the following types :

Recurring decimals : If in a decimal fraction, a figure or a set of figures is repeated continually, then such a number is called a recurring decimal.

If a single figure is repeated, it is shown by putting a dot on it. But if a set of a figures is repeated, we express it either by putting one dot at the starting digit and one dot at the last digit of the repeating digits or by placing a bar or a vinculum on the repeating digit (s).

(i) $2/3 = 0.6666\ldots = 0.\dot{6} = 0.\overline{6}$

(ii) $22/7 = 3.142857142857 = 3.\dot{1}42857\dot{} = 3.142857\dot{} = 3.\overline{142857}$

(iii) $95/6 = 15.83333\ldots = 15.8\dot{3} = 15.8\overline{3}$

Pure recurring decimals : A decimal in which all figures after the decimal point repeat is called a pure recurring decimal.

Ex. $0.\dot{6}$, $3.\dot{1}42857\dot{}$ etc.

Conversion of a pure recurring decimal into fraction

Rule : Write the recurring figures only once in the numerator and take as many nines in the denominator as the number of repeating figures.

(1) $0.\overline{6} = 6/9 = 2/3$.

(2) $16.\dot{6} = 16 + 0.\overline{6} = 16 + 6/9 = 16 + 2/3 = 50/3$.

To convert a mixed recurring decimal into fraction

$$\left(\frac{p}{q}\right)\text{form} = \frac{(\text{Complete numbers}) - (\text{number formed by Non-repeating digit})}{\text{No. of 9 as no. of repeating digits after that write no. of 0 as no. of non repeating digits.}}$$

e.g. (i) $0.\overline{35} = \frac{35-0}{99} = \frac{35}{99}$ (ii) $0.4\overline{35} = \frac{435-4}{990} = \frac{431}{990}$

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1.8 H.C.F & L.C.M of numbers

- **The least number which when divided by d_1, d_2 and d_3 leaves the remainders r_1, r_2 and r_3 respectively, such that $(d_1-r_1) = (d_2-r_2) = (d_3-r_3)$ is (L.C.M. of d_1, d_2 and d_3) – (d_1-r_1) or (d_2-r_2) or (d_3-r_3).**

NUMERICAL CHALLENGE 1.7

Find the least number which when divided by 9, 10 and 15 leaves the remainders 4, 5 and 10, respectively.

Solution

Here, 9–4 = 10–5=15–10=5

Also, L.C.M. (9, 10, 15)= 90

∴ the required least number = 90–5=85.

- **A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder is $(d_1 \times r_2 + r_1)$.**

NUMERICAL CHALLENGE 1.8

A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.

Solution

The required remainder

$= d_1 \times r_2 + r_1 = 10 \times 7 + 5 = 75.$

- **To find the number of numbers divisible by a certain integer.**

NUMERICAL CHALLENGE 1.9

(i) How many numbers up to 532 are divisible by 15 ?

Solution

We divide 532 by 15.

$532 = 35 \times 15 + 7$

The quotient obtained is the required number of numbers. Thus, there are 35 such numbers.

(ii) How many numbers up to 300 are divisible by 5 and 7 together ?

Solution

L.C.M. of 5 and 7 = 35

We divide 300 by 35

$300 = 8 \times 35 + 20$

Thus there are 8 such numbers.

- **Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. The divisor is given by $r_1 + r_2 - r_3$.**

NUMERICAL CHALLENGE 1.10

Two numbers when divided by a certain divisor give remainders 473 and 298, respectively. When their sum is divided by the same divisor, the remainder is 236. Find the divisor.

Solution

The required divisor

$= 437 + 298 - 236 = 499.$

- **Product of two numbers = L.C.M. of the numbers × H.C.F. of the numbers.**

NUMERICAL CHALLENGE 1.11

The H.C.F. and the L.C.M. of any two numbers are 63 and 1260, respectively. If one of the two numbers is 315, find the other number.

Solution

The required number $= \frac{\text{L.C.M.} \times \text{H.C.F.}}{\text{First number}} = \frac{1260 \times 63}{315} = 252$

- **To find the greatest number that will exactly divide x, y and z.**
  Required number = H.C.F. of x, y, and z.

NUMERICAL CHALLENGE 1.12

Find the greatest number that will exactly divide 200 and 320.

Solution

The required greatest number

= H.C.F. of 200 and 320 = 40.

- **To find the greatest number that will divide x, y, and z leaving remainders a, b, and c, respectvely.**
  Required number = H.C.F. of (x–a), (y–b) and (z–c).

NUMERICAL CHALLENGE 1.13

Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11, respectively.

Solution

The required greatest number

= H.C.F. of (148–4), (246–6) and (623–11),

i.e. H.C.F. of 144, 240 and 612 = 12.

- **To find the least number which is exactly divisible by x, y and z.**
  Required number = L.C.M. of x, y and z.

NUMERICAL CHALLENGE 1.14

What is the smallest number which is exactly divisible by 36, 45, 63 and 80?

Solution

The required smallest number

= L.C.M. of 36, 45, 63 and 80 = 5040.

- **To find the least number which when divided by x,y, and z leaves the remainders a,b, and c, respectively. It is a always observed that (x–a) = (y–b)=(z–c) = k(say)**

∴ **Required number = (L.C.M. of x, y, and z)–k.**

NUMERICAL CHALLENGE 1.15

Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53, respectively.

Solution

Since (36–25) = (48–37) = (64–53) = 11, therefore the required smallest number

= (L.C.M. of 36, 48 and 64)–11

= 576–11 = 565.

- **To find the least number which when divided by x, y and z leaves the same remainder r in each case.**
  Required number = (L.C.M. of x, y, and z) +r.

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NUMERICAL CHALLENGE 1.16

Find the least number which when divided by 12, 16 and 18, will leave in each case a remainder 5.

Solution

The required smallest number

= (L.C.M of 12, 16 and 18) + 15

= 144 + 5 = 149.

- **To find the greatest number that will divide x, y, and z leaving the same remainder in each case.**

  (a) When the value of remainder r is given :

  Required number = H.C.F. of (x–r), (y–r) and (z–r).

  (b) When the value of reminder is not given:

  Required number = H.C.F. of |(x–y)|, |(y–z)| and |(z–x)|

NUMERICAL CHALLENGE 1.17

(a) Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.

Solution

The required greatest number

= H.C.F. of (772 – 5) and (2778 – 5)

= H.C.F. of 767 and 2773

= 59.

(b) Find the greatest number which on dividing 152, 277 and 427 leaves remainder.

Solution

The required greatest number.

= H.C.F. of |(x – y)|, |(y – z) and (z – x)|

= H.C.F. of |(152 – 277)|, |(277 – 427)|

and |(427 – 152)|

= H.C.F. of 125, 275 and 150

= 25.

- **To find the n-digit greatest number which, when divided by x, y and z.**

  (a) leaves no remainder (i.e., exactly divisible)

  Step 1 – L.C.M. of x, y and z = L

  Step 2 – $L\overline{)\text{n - digit greatest number}}$, **Remainder = R**

  Step 3 – Required number = n-digit greatest number – R

  (b) leaves reminder K in each case

  Required number = (n-digit greatest number–R) + K.

NUMERICAL CHALLENGE 1.18

Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28, leaves 3 as a remainder in each case.

Solution

L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)9999}(39$$
$$\underline{9828}$$
$$171$$

∴ The required number = (9999–171)+3= 9931.

- **To find the n-digit smallest number which when divided by x, y and z**

  (a) leaves no remainder (i.e. exactly divisible)

  Step 1 – L.C.M. of x, y and z= L

  Step 2 – $L\overline{)\text{n-digit smallest number}}$, **Remainder= R**

  Step 3 – Required number = n-digit smallest number + (L–R).

  (b) leaves remainder K in each case.

  Required number = n-digit smallest number + (L–R) + k.

NUMERICAL CHALLENGE 1.19

(a) Find the least number of four digits which is divisible by 4, 6, 8 and 10.

Solution

L.C.M. of 4, 6, 8, and 10 = 120.

$$120\overline{)1000}(8$$
$$\underline{960}$$
$$40$$

∴ The required number = 1000 + (120 – 40) = 1080.

(b) Find the smallest 4-digit number, such that when divided by 12, 18, 21 and 28, it leaves remainder 3 in each case.

Solution

L.C.M. of 12, 18, 21 and 28 = 252.

$$252\overline{)1000}(3$$
$$\underline{756}$$
$$244$$

∴ The required number

= 1000+(252–244)+3

= 1011.

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1.9 Number of factors of a given number

Number of factors (or divisors) of a given number (composite number)

Let us assume a composite number say 24 then find the number of factors.

$24 = 1 \times 24$
2×12
3×8
4×6

We see that there are total 8 factors namely,

1, 2, 3, 4, 6, 8, 12 and 24.

Let there be a composite number N and its prime factors be a, b, c, d,..... etc and p,q, r,s...etc. be the indices (or powers) of the a, b, c, d..... etc. respectively i.e., if N can be expressed as

$N = a^p . b^q . c^r . d^s ...$

then, the number of total divisors or factors of N is

$(p+1)(q+1)(r+1)(s+1).....$

NUMERICAL CHALLENGE 1.20

Find the total number of factors of 540.

(1) 24 (2) 20 (3) 30 (4) None of these

Solution

$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$540 = 2^2 \times 3^3 \times 5^1$

Therefore total number of factors of 540 is

$(2+1)(3+1)(1+1) = 24$

1.10 Sum of factors of given number

Once again if you want to find the sum of smaller composite numbers, then you can do it manually, but for larger numbers its a problem.

e.g. Sum of factors of 24

$= 1 + 2 + 3 + 4 + 6 + 8 + 12 + 24 = 60$

Let N be the composite number and a, b, c, d.. be its prime factors and p,q, r,s be the indices (or powers) of a,b,c,d i.e., if N can be expressed as

$N = a^p . b^p . c^r . d^s ...$

then the sum of all the divisors (or factors) of $N = \frac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)(d^{s+1}-1)}{(a-1)(b-1)(c-1)(d-1)}$

NUMERICAL CHALLENGE 1.21

Find the sum of factors of 270.

Solution

$270 = 2 \times 3^3 \times 5$

$\therefore$ Sum of factors of 270 $= \frac{(2^{1+1}-1)(3^{3+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$

$= \frac{3 \times 80 \times 24}{1 \times 2 \times 4} = 720$

1.11 Product of factors

Let us assume a very small number 24 and see the factors

$24 = 1 \times 24$
$= 2 \times 12$
$= 3 \times 8$
$= 4 \times 6$

Note, it is obvious from the above explanation that the product of factors of 24

$= (1 \times 24) \times (2 \times 12) \times (3 \times 8) \times (4 \times 6)$

$= 24 \times 24 \times 24 \times 24 = (24)^4$

Thus, **the product of factors of composite number $N = N^{n/2}$, where n is the total number of factors of N.**

NUMERICAL CHALLENGE 1.22

Find the product of divisors of 7056.

(1) $(84)^{48}$ (2) $(84)^{44}$ (3) $(84)^{45}$ (4) None of these

Solution

$\therefore 7056 = 2^4 \times 3^2 \times 7^2 \Rightarrow (84)^2$

$\therefore$ Number of factors /divisors of 7056

$= (4+1)(2+1)(2+1) = 45$

$\therefore$ product of factors $= (7056)^{45/2} = (84)^{45}$

Hence (c) is the correct option.

1.12 Number of odd factors of given number

Let us assume a smaller number e.g.,, 24

$24 = 1 \times 24$ also $24 = 2^3 \times 3^1$
$= 2 \times 12$
$= 3 \times 8$

We can see that there are total 2 odd factors namely 1 and 3.

Further assume another number say 36

$36 = 1 \times 36$ also $36 = 2^2 \times 3^2$
$= 2 \times 18$
$= 3 \times 12$
$= 4 \times 9$
$= 6 \times 6$

So, we can see that there are only 3 odd factors viz., 1, 3 and 9.

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Once again we assume another number say 90

then $90 = 1 \times 90$ also $90 = 2 \times 3^2 \times 5$

$= 2 \times 45$

$= 3 \times 30$

$= 5 \times 18$

$= 6 \times 15$

$= 9 \times 10$

Thus there are only 6 odd factors namely 1, 3, 5, 9, 15, 45.

To get the number of odd factors of a numbers N first of all express the number N as

$N = (p_1^a \times p_2^b \times p_3^c \times \ldots) \times (e^n)$

(where, $p_1, p_2, p_3, \ldots$ are the odd prime factors and e is the even prime factor)

Then the total number of odd factors

$= (a+1)(b+1)(c+1)\ldots$

NUMERICAL CHALLENGE 1.23

Find the number of odd factors of 36.

Solution

$36 = 2^2 \times 3^2$

$\therefore$ Number of odd factors $= (2 + 1) = 3$

1.13 Number of even factors a composite number

Number of even factors of a number

= (Total number of factors of the given number) – (Total number of odd factors)

NUMERICAL CHALLENGE 1.24

(a) Find the number of even factors (or daisors) of 24.

Solution

number of even factors $= 8 - 2 = 6$

(b) Find the number of even factors of 36

Solution

number of even factors $= 9 - 3 = 6$

(c) Find the number of even factors of 90

Solution

number of even factors $= 12 - 6 = 6$

1.14 Number of ways of expressing a composite number as a product of two factors

Let us consider an example of small composite number say, 24

then $24 = 1 \times 24$

2×12

3×8

4×6

So it is clear that

the number of ways of expressing a composite no. as a product of two factors $= \frac{1}{2} \times$ **the no. of total factors**

NUMERICAL CHALLENGE 1.25

Find the number of ways of expressing 180 as a product of two factors.

Solution

$180 = 2^2 \times 3^2 \times 5^1$

Number of factors $= (2 + 1)(2 + 1)(1 + 1) = 18$

Hence, there are total $\frac{18}{2} = 9$ ways in which 180 can be expressed as a product of two factors.

• **Note** - As you know when you express any perfect square number 'N' as a product of two factors namely $\sqrt{N}$ and $\sqrt{N}$, and also know that since in this case $\sqrt{N}$ appears two times but it is considered only once while calculating the no. of factors so we get always an odd number as number of factors so we can not divide the odd number exactly by 2 as in the above formula. So if we have to consider these two same factors then we find the number of ways of expressing N as a **product of two factors** $= \frac{\text{(Number of factors + 1)}}{2}$.

Again if it is asked that find no. of ways of expressing N as a product of two distinct factors then we do not consider 1 way (i.e. $N = \sqrt{N} \times \sqrt{N}$) then **no. of ways** $= \frac{\text{(Number of factors – 1)}}{2}$

NUMERICAL CHALLENGE 1.26

(a) Find the number of ways of expressing 36 as a product of two factors.

Solution

$36 = 2^2 \times 3^2$

Number of factors $= (2+1)(2+1) = 9$

Hence the no. of ways of expressing 36 as a product of two factors $= \frac{(9+1)}{2} = 5$

as $36 = 1 \times 36, 2 \times 18, 3 \times 12, 4 \times 9$ and 6×6

(b) In how many ways can 576 be expressed as the product of two distinct factors?

Solution

$576 = 2^6 \times 3^2$

$\therefore$ Total number of factors $= (6 + 1)(2 + 1) = 21$

So the number of ways of expressing 576 as a product of two distinct factors $= = \frac{21-1}{2} = 10$

• **Note** – Since the word 'distinct' has been used therefore we do not include $576 = 26 \times 26$.

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1.15 Cyclicity

We are having 10 digits in our number systems and some of them shows special characteristics like they, repeat their unit digit after a cycle, for example 1 repeat its unit digit after every consecutive power. So its cyclicity is 1 on the other hand digit 2 repeat its unit digit after every fourth power, hence the cyclicity of 2 is four. The cyclicity of digits are as follows

Digit	Cyclicity
0, 1, 5 and 6	1
4 and 9	2
2, 3, 7 and 8	4

So, if we want to find last digit of 2^{45}, divide 45 by 4. The remainder is 1 so the last digit of 2^{45} would be same as the last digit of 2^1 which is 2.

To Find the Unit Digit in Exponential Expression:

(i) Where there is 2 in unit's place of any number.

Since, in 2^1 unit digit is 2, in 2^2 unit digit is 4, in 2^3 unit digit is 8, in 2^4 unit digit is 6, after that the unit's digit repeats e.g. unit digit $(12)^{12}$ is equal to the unit digit of 2^4 i.e. 6

Ex. In $(32)^{33}$ unit digit is equal to the unit digit of 2^1 i.e. 2.

(ii) When there is 3 in unit's place of any number.

Since, in 3^1 unit digit is 3, in 3^2 unit digit is 9, in 3^3 unit digit is 7, in 3^4 unit digit is 1, after that the unit's digit repeats.

Ex. In $(43)^{46}$ unit digit is 9.

(iii) When there is 4 in unit's place of any number.

Since, in 4^1 unit digit is 4, in 4^2 unit digit is 6, after that the unit's digit repeats.

Ex. In $(34)^{14}$ unit digit is 6.

Ex. In $(34)^{33}$ unit digit is 4.

(iv) When there is 5 in unit's place of any number.

Since , in 5^1 unit digit is 5, in 5^2 unit digit is 5 and so on.

Ex. In $(25)^{15}$ unit digit is 5.

(v) When there is 6 in unit's place of any number.

Since, in 6^1 unit digit is 6, in 6^2 unit digit is 6 & so on.

Ex. In $(46)^{13}$ unit digit is 6.

(vi) When there is 7 in unit's place of any number.

Since, in 7^1 unit digit is 7, in 7^2 unit digit is 9, in 7^3 unit digit is 3, in 7^4 unit digit is 1, after that the unit's digit repeats.

Ex. In $(57)^9$ unit digit is 7.

Ex. In $(97)^9$ unit digit is 3.

(vii) When there is 8 in unit's place of any number.

Since, in 8^1 unit digit is 8, in 8^2 unit digit is 4, in 8^3 unit digit is 2, in 8^4 unit is 6, after that unit's digit repeats after a group of 4.

(viii) When there is 9 in unit's place of any number.

Since, in 9^1 unit digit is 9, in 9^2 unit digit is 1, after that unit digit repeats after a group of 2.

(ix) When there is zero in unit's place of any number.

There will always be zero in unit's place.

NUMERICAL CHALLENGE 1.27

(a) Find the last digit of

(i) 3^{57} (ii) 13^{59}

Solution

(i) The cyclicity of 3 is 4. Hence, $\frac{57}{4}$ gives the remainder 1. So the last digit of 3^{57} is same as the last digit of 3^1, i.e. 3.

(ii) The number of digits in the base will not make a difference to the last digit. It is last digit of the base which decides the last digit of the number itself. For 13^{59}, we find $\frac{59}{4}$ which gives a remainder 3. So the last digit of 13^{59} is same as the last digit of 3^3, i.e 7.

(b) Find the last digit of the product $7^{23} \times 8^{13}$.

Solution

Both 7 and 8 exhibit a cyclicity of 4, the last digit are

$7^1 = 7$	$8^1 = 8$
$7^2 = 9$	$8^2 = 4$
$7^3 = 3$	$8^3 = 2$
$7^4 = 1$	$8^4 = 6$
$7^5 = 7$	$8^5 = 8$

The cycle would repeat itself for higher powers. 7^{23} ends with the same last digit as 7^3, i.e. 3.
8^{13} ends with the same last digit as 8^1, i.e. 8. Hence, the product of the two numbers would end with the same last digit as that of 3×8, i.e. 4.

(c) Find unit's digit in $y = 7^{17} + 7^{34}$

Solution

$7^{17} + 7^{34} = 7^1 + 7^2 = 56$, hence the unit digit is 6.

(d) What will be the last digit of $(73)^{75^{76}}$?

Solution

Let $(73)^{75^{76}} = (73)^x$ where $x = 75^{76} = (75)^{\text{even power}}$

$\because$ Cyclicity of 3 is 4

$\therefore$ To find the last digit we have to find the remainder when x is divided by 4.

$x = (75)^{\text{even power}} = (76-1)^{\text{even power}}$, where n is dvided by 4 so remainder will be 1.

Therefore, the last digit of $(73)^{75^{76}}$ will be $3^1 = 3$.

1.16 Euclid's division lemma

Euclid's division lemma states that "For any two positive integers a and b, there exist integers q and r such that $a = bq + r$, $0 \le r < b$."

e.g.(i) Consider number 23 and 5, then :

$23 = 5 \times 4 + 3$

Comparing with $a = bq + r$

we get, $a = 23$, $b = 5$, $q = 4$, $r = 3$ and $0 \le r < b$ (as $0 < 3 < 5$)

(ii) Consider positive integers 18 and 4

$18 = 4 \times 4 + 2$

For 18 (= a) and 4 (= b) we have $q = 4$, $r = 2$ and $0 \le r < b$

In the relation $a = bq + r$, where $0 \le r < b$ is nothing but a statement of the long division of number a by b in which q is the quotient obtained and r is the remainder.

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1.17 Euclid's division algorithm

In mathematics, the Euclid's Algorithm is an efficient method for computing the greatest common divisor (GCD) or highest common factor (HCF).

So, let us state Euclid's division algorithm clearly.

To obtain the HCF of two positive integers, say c and d, with c > d follow the steps below :

Step-1 : Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.

Step-2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step-3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because HCF (c,d) = HCF (d,r) where the symbol HCF (c,d) denotes the HCF of c and d.

Ex. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

Solution

In order to find the HCF of 441, 567 & 693, we first find the HCF of 441 & 567 by Euclid's division algorithm.

Using division algorithm, we get

$567 = 441 \times 1 + 126$

$441 = 126 \times 3 + 63$

$126 = 63 \times 2 + 0$

So, HCF (567, 441) = 63

Now, we find the HCF of 63 and 693

$693 = 63 \times 11 + 0$

∴ HCF (63, 693) = 63

Hence, HCF (441, 567, 693) = 63

1.18 Fundamental theorem of arithmetic

The fundamental theorem of arithmetic (FTA) tells us something important about the relationship between composite numbers and prime numbers. It is usually stated as follows :

"Every composite number can be expressed as a product of primes, and their decomposition is unique, a part from the order in which the prime factors occur."

e.g. $12600 = 2^3.3^2.5^2.7$

Thus we have expressed the composite number 12600 as product of powers of primes in ascending order and this decomposition is unique.

1.19 Points to remember

- For any three positive integers p,q,r, HCF (p,q,r) × LCM (p,q,r) ≠ p × q × r. However, the following results hold good. $\text{LCM}(p,q,r) = \frac{p.q.r\ \text{HCF}(p,q,r)}{\text{HCF}(p,q) \cdot \text{HCF}(q,r) \cdot \text{HCF}(p,r)}$
- All odd numbers can be obtained by the formula, $f(n) = 2n + 1, n = 1,2,3,....$ while even numbers can be obtained by the formula, $g(n) = 2n, n = 1,2,3,....$
- L.C.M. of pair of coprimes = Product of co-primes

EXERCISE-1

1. A rational number can be expressed as a terminating decimal if the denominator has factors
(1) 2 or 5 (2) 2, 3 or 5 (3) 3 or 5 (4) None of these

2. The only prime number which is even is
(1) 2 (2) 4 (3) 6 (4) none of these

3. The value of $23.\overline{43} + 5.\overline{2}$ is
(1) $\frac{2395}{990}$ (2) $\frac{2527}{99}$ (3) $\frac{5169}{990}$ (4) $\frac{2837}{99}$

4. If $2 = x + \cfrac{1}{1+\cfrac{1}{3+\frac{1}{4}}}$, then value of x is
(1) $\frac{12}{17}$ (2) $\frac{13}{17}$ (3) $\frac{18}{17}$ (4) $\frac{21}{17}$

5. If R : Every fraction is a rational number and T : Every rational number is a fraction, then which of the following is correct?
(1) R is True and T is False. (2) R is False and T is True.
(3) Both R and T are True. (4) Both R and T are False.

6. $5.\overline{2}$ is equal to
(1) $\frac{45}{9}$ (2) $\frac{46}{9}$ (3) $\frac{47}{9}$ (4) None of these

7. For any two rational numbers A and B, which of the following properties are correct?
(i) A < B (ii) A = B (iii) A > B
(1) Only (i) and (ii) are correct. (2) Only (ii) and (iii) are correct.
(3) Only (ii) is correct. (4) All (i), (ii), (iii) are correct.

8. If $x = \frac{3+\sqrt{2}}{3-\sqrt{2}}$ and $y = \frac{3-\sqrt{2}}{3+\sqrt{2}}$, the value of (x + y) is
(1) $\frac{306}{49}$ (2) $\frac{484}{49}$ (3) $\frac{22}{7}$ (4) $\frac{73}{7}$

9. If x : Every whole number is a natural number and y : 0 is not a natural number, then which of the following statement is true?
(1) x is false and y is the correct explanation of x.
(2) x is true and y is the correct explanation of x.
(3) x is true and y is false.
(4) Both x and y are true.

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10. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^2 + b^2$ is

(1) 34 (2) 35 (3) 36 (4) 37

11. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then the value of $a^3 + b^3$ is

(1) 194 (2) 196 (3) 198 (4) 200

12. If $x = (7+4\sqrt{3})$, then the value of $x^2 + \frac{1}{x^2}$ is

(1) 193 (2) 194 (3) 195 (4) 196

13. If $x = 7 + 4\sqrt{3}$, then the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is

(1) 8 (2) 6 (3) 5 (4) 4

14. $8 - 8 \times \frac{2\frac{1}{5} - 1\frac{2}{7}}{2 - \frac{1}{6 - \frac{1}{6}}}$ is equal to

(1) 6 (2) 2 (3) 4 (4) 8

15. The rational form of $2.74\overline{35}$ is

(1) $\frac{27161}{9999}$ (2) $\frac{27}{99}$ (3) $\frac{27161}{9900}$ (4) $\frac{27161}{9000}$

16. If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, then the value of 'a' and 'b' is

(1) a = 2, b = – 1 (2) a = 2, b = 1
(3) a = – 2, b = 1 (4) a = – 2, b = – 1

17. The value of $\frac{1}{\sqrt{3}+\sqrt{2}-1}$ on simplifying upto 3 decimal places, given that $\sqrt{2} = 1.4142$ and $\sqrt{6} = 2.4495$ is

(1) 0.166 (2) 0.366 (3) 0.466 (4) 0.566

18. π is

(1) Rational (2) Irrational (3) Imaginary (4) An integer

19. An irrational number is

(1) A terminating and non-repeating decimal
(2) A non-terminating and non-repeating decimal
(3) A terminating and repeating decimal
(4) A non-terminating and repeating decimal

20. Which of the following statements is true ?

(1) Every point on the number line represents a rational number
(2) Irrational numbers cannot be represented by points on the number line
(3) $\frac{22}{7}$ is a rational number
(4) None of these

21. The sum of rational and irrational number is always

(1) Rational (2) Irrational (3) Both (4) Can't say

22. The product of rational and irrational number is always

(1) Rational (2) Irrational (3) Both (4) Can't say

23. Rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(1) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (2) $\frac{\sqrt{2}\times\sqrt{3}}{2}$ (3) 1.5 (4) 1.8

24. The number $\frac{5-\sqrt{5}}{5+\sqrt{5}}$ is

(1) Rational (2) Irrational (3) Both (4) Can't say

25. $0.\overline{23} + 0.\overline{22} = ?$

(1) $0.\overline{45}$ (2) $0.\overline{43}$ (3) $0.4\overline{5}$ (4) 0.45

26. $0.\overline{018}$ can be expressed in the rational form as

(1) $\frac{18}{1000}$ (2) $\frac{18}{990}$ (3) $\frac{18}{9900}$ (4) $\frac{18}{999}$

27. The equivalent rational form of $17.\overline{6}$ is

(1) $\frac{53}{3}$ (2) $\frac{88}{5}$ (3) $\frac{44}{25}$ (4) None of these

28. $\frac{961}{625}$ is a

(1) Terminating decimal (2) Nonterminating decimal
(3) Cannot be determined (4) None of these

29. When simplified, the product $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right).....\left(1-\frac{1}{n}\right)$ equals :

(1) $\frac{1}{n}$ (2) $\frac{2}{n}$ (3) $\frac{2(n-1)}{n}$ (4) $\frac{2}{n(n+1)}$

30. Which of the following has most number of divisors ?

(1) 99 (2) 101 (3) 176 (4) 182

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31. A number n is said to be perfect if the sum of all its divisors (excluding n itself) is equal to n. An example of perfect number is
(1) 6 (2) 9 (3) 15 (4) 21

32. The H.C.F. of $2^2 \times 3^3 \times 5^5$, $2^3 \times 3^2 \times 5^2 \times 7$ and $2^4 \times 3^4 \times 5 \times 7^2 \times 11$ is
(1) $2^2 \times 3^2 \times 5$ (2) $2^2 \times 3^2 \times 5 \times 7 \times 11$ (3) $2^4 \times 3^4 \times 5^5$ (4) $2^4 \times 3^4 \times 5^5 \times 7 \times 11$

33. Which of the following is a pair of co-primes ?
(1) (16, 62) (2) (18, 25) (3) (21, 35) (4) (23, 92)

34. The L.C.M. of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^4 \times 5^2 \times 7$ and $2^5 \times 3^3 \times 5^3 \times 7^2 \times 11$ is
(1) $2^3 \times 3^2 \times 5$ (2) $2^5 \times 3^4 \times 5^3$ (3) $2^3 \times 3^2 \times 5 \times 7 \times 11$ (4) $2^5 \times 3^4 \times 5^3 \times 7^2 \times 11$

35. The least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3 is
(1) 3 (2) 13 (3) 23 (4) 33

36. The least number which is a perfect square and is divisible by each of the numbers 16, 20 and 24, is
(1) 1600 (2) 3600 (3) 6400 (4) 14400

37. The smallest number which when diminished by 7, is divisible by 12, 16, 18, 21 and 28 is
(1) 1008 (2) 1015 (3) 1022 (4) 1032

38. The least number which when increased by 5 is divisible by each one of 24, 32, 36 and 54, is
(1) 427 (2) 859 (3) 869 (4) 4320

39. The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8, is
(1) 504 (2) 536 (3) 544 (4) 548

40. The largest four-digit number which when divided by 4, 7 or 13 leaves a remainder of 3 in each case, is
(1) 8739 (2) 9831 (3) 9834 (4) 9893

41. The least multiple of 7, which leaves a remainder of 4, when divided by 6, 9, 15 and 18 is
(1) 74 (2) 94 (3) 184 (4) 364

42. The least number, which when divided by 48, 60, 72, 108 and 140 leaves 38, 50, 62, 98 and 130 as remainders respectively, is
(1) 11115 (2) 15110 (3) 15120 (4) 15210

43. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves remainders 7, 10 and 13 respectively.
(1) 3002 (2) 3013 (3) 3024 (4) 3036

44. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder, is
(1) 1677 (2) 1683 (3) 2523 (4) 3363

45. The product of two numbers is 960. If H.C.F. is 8, then the numbers are
(1) 24,40 (2) 8, 120 or 24, 40 (3) 8, 140 (4) none of these

46. The least number divisible by 12, 15, 20, and is perfect square is
(1) 900 (2) 400 (3) 36 (4) 256

47. The largest number which divides 133 and 245 leaving a remainder 5 is
(1) 17 (2) 15 (3) 8 (4) 16

48. The H.C.F. of two numbers is $\frac{1}{5}$ of their L.C.M. If the product of two number is 720, then the H.C.F. of the numbers is
(1) 13 (2) 12 (3) 14 (4) 18

49. The L.C.M. of two numbers is 39780 and their ratio is 13 : 15, then the numbers are
(1) 273,315 (2) 2652,3060 (3) 516, 685 (4) None of these

50. The L.C.M. of two numbers is 14 times of their H.C.F. The sum of L.C.M. and H.C.F. is 600. If one of the number is 80, then other is
(1) 280 (2) 218 (3) 25 (4) 45

51. Four bells begin to toll together and toll respectively at intervals of 5, 6, 8 and 12 seconds. How many times will they toll together in an hour excluding the one at the start
(1) 30 (2) 19 (3) 13 (4) 5

52. Two ropes of length 28 m and 36 m are to be cut into bits of same length. The greatest possible length of each is
(1) 7 (2) 3 (3) 4 (4) 5

53. There are 28 mango trees, 42 apple trees and 21 orange trees. They have to be planted in rows such that each row contains the same number of trees of one variety only. The minimum number of rows in which the above trees may be planted
(1) 13 (2) 12 (3) 11 (4) 10

54. A heap is to be formed with lots of 8, 10 and 15 pebbles of different colours. The smallest number of pebbles in the heap is
(1) 121 (2) 120 (3) 110 (4) 8

55. Three measuring tapes are 64 cm, 72 cm, 96 cm, the shortest length that can measured by any one of the tapes exactly is
(1) 576 cm (2) 120 cm (3) 8 cm (4) 10 cm

56. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is
(1) 28 (2) 32 (3) 40 (4) 64

57. $\frac{(x^{a+b})^2 (x^{b+c})^2 (x^{c+a})^2}{(x^a . x^b . x^c)^4} =$
(1) –1 (2) 0 (3) 1 (4) None

58. $\frac{2^{n+4} - 2(2^n)}{2(2^{n+3})} + 2^{-3}$ is equal to
(1) 2^{n+1} (2) $-2^{n+1} + \frac{1}{8}$ (3) $\frac{9}{8} - 2^n$ (4) 1

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59. The value of $\left[(x^{a-b})^{\frac{1}{a-c}}\right]^{\frac{a}{a-c}}$ =

(1) x (2) 1/x (3) x^a (4) $1/x^a$

60. If $a = \frac{2+\sqrt{3}}{2-\sqrt{3}}$, $b = \frac{2-\sqrt{3}}{2+\sqrt{3}}$, then the value of a + b is

(1) 14 (2) −14 (3) $8\sqrt{3}$ (4) $-\sqrt{3}$

61. If the product of two irrational numbers is rational, then which of the following can be concluded ?

(1) The ratio of the greater and the smaller numbers is an integer

(2) The sum of the numbers must be rational

(3) The excess of the greater irrational number over the smaller irrational number must be rational

(4) None of these

62. The LCM and HCF of two numbers are equal, then the numbers must be

(1) Prime (2) Co-prime (3) Composite (4) Equal

63. Find the remainder when the square of any prime number greater than 3 is divided by 6.

(1) 1 (2) 3 (3) 2 (4) 4

64. What is the digit in the tens place in the product of the first 35 even natural numbers ?

(1) 6 (2) 2 (3) 0 (4) 5

65. Find the unit's digit in the product of the first 50 odd natural numbers.

(1) 0 (2) 5 (3) 7 (4) None

66. There are 20 balls. The balls are numbered consecutively starting from anyone of the numbers from 1 to 20. For any case, the sum of the numbers on all the balls will be a/an

(1) odd number (2) even number (3) prime number (4) Cannot say

67. Which pair of numbers below are twin primes ?

(1) 8 and 9 (2) 2 and 3 (3) 3 and 7 (4) 41 and 43

68. What is the number in the units place of $(763)^{84}$?

(1) 1 (2) 3 (3) 7 (4) 9

69. If the numbers a − b and a + b are twin primes, then a and b are necessarily

(1) Twin primes (2) Co-primes (3) Cannot say (4) None

70. Find the units digit in the expansion of

$(44)^{44} + (55)^{55} + (88)^{88}$

(1) 7 (2) 5 (3) 4 (4) 3

71. Find the digit in the units place of $(676)^{99}$.

(1) 9 (2) 2 (3) 4 (4) 6

72. The greatest five digit number exactly divisible by 9 and 13 is

(1) 99945 (2) 99918 (3) 99964 (4) 99972

73. If the number 2345p60q is exactly divisible by 3 and 5, then the maximum value of p + q is

(1) 12 (2) 13 (3) 14 (4) 15

74. If x > 0, y > 0 and z > 0 and $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$, then

(1) $x^3+y^3+z^3 \geq 3xyz$ (2) $x^3+y^3+z^3 \geq 6xyz$ (3) $x^3+y^3+z^3>3xyz$ (4) $x^3+y^3+z^3<3xyz$

75. If $\frac{1}{x+1}+\frac{2}{y+2}+\frac{2006}{z+2006}=1$, find the value of $\frac{x^2}{x^2+x}+\frac{y^2}{y^2+2y}+\frac{z^2}{z^2+2006z}$

(1) 2 (2) 14 (3) 3 (4) 0

76. Simplify : $\frac{(\sqrt[4]{625^{-3}} \times \sqrt[3]{0.00001} \times \sqrt[3]{729} \times \sqrt[3]{(0.008)^{-2}})}{(\sqrt[5]{243} \times \sqrt[3]{125^{-3}})}$

(1) $\frac{5}{12}$ (2) $\frac{15}{2}$ (3) $\frac{3}{400}$ (4) 5

77. Find the value of $\left(\frac{117649}{1771561}\right)^{1/6}$

(1) 3/11 (2) 13/7 (3) 7/11 (4) 7/13

78. If the product of two surds is a rational number, then each one of them is called the ____ of the other.

(1) Negative (2) Rationalising factor (3) Conjugate (4) Binomial surd

79. The value of $\sqrt[l+m]{\frac{x^l}{x^m}} \cdot \sqrt[m+n]{\frac{x^m}{x^n}} \cdot \sqrt[n+l]{\frac{x^n}{x^l}}$ = ____.

(1) x (2) −1 (3) 1 (4) 0

80. The value of $\left[\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}\right]$ = ?

(1) 3(a+b)(b+c)(c+a) (2) 3(a−b)(b−c)(c−a) (3) (a+b)(b+c)(c+a) (4) 1

81. The sum of a number and its reciprocal is thrice the difference of the number and its reciprocal. The number is ____.

(1) $\pm\sqrt{2}$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\sqrt{3}$ (4) Both (1) and (2)

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82. Solve for x : $\frac{2^{x-3}}{8^{-x}} = \frac{32}{4^{(1/2)x}}$

(1) $2\frac{1}{5}$ (2) $1\frac{1}{5}$ (3) $3\frac{1}{5}$ (4) $1\frac{3}{5}$

83. $2.\overline{6} - 0.\overline{82} =$

(1) $\frac{182}{99}$ (2) $\frac{1820}{999}$ (3) $\frac{175}{99}$ (4) $\frac{184}{999}$

84. Find the missing value : $\frac{(13)^3 + 7^3}{(13)^2 + 7^2 - ?} = 20$

(1) 6 (2) 20 (3) 91 (4) 19

85. Value of $\frac{a^3 + b^3 + c^3 - 3abc}{ab + bc + ca - a^2 - b^2 - c^2}$, when a = –5, b = –6, c = 10 is

(1) 1 (2) –1 (3) 2 (4) –2

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	1	1	4	4	1	3	4	3	1	1	3	2	4	3	3
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	1	3	2	2	4	2	3	3	2	1	4	1	1	2	3
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	1	1	2	4	3	2	2	2	4	2	4	2	2	2	2
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	4	2	2	1	1	3	1	2	1	3	3	4	1	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	4	4	1	3	2	2	4	1	2	1	4	2	2	1	1
Que.	76	77	78	79	80	81	82	83	84	85					
Ans.	2	3	2	3	3	1	4	1	3	1					

CHAPTER 2 Surds & Indices

2.1 Radicals

A radical expression (or simply a radical) is an expression of the type $\sqrt[n]{x}$. The sign '$\sqrt{\ }$' is called the radical sign. The number under this sign, i.e., 'x' is called the radicand and the number in the angular part of the sign, i.e., 'n' is the order of the radical. At present, we shall deal only with cases where x is a real number. Depending on the values of x, n can have certain corresponding values. Initially we shall consider only positive integral values of n. But remember that $\sqrt[n]{x} = x^{1/n}$. We have the following results for radicals.

If x > 0, n can have real value except zero.
If x = 0, n can have any positive real value.
If x < 0, n can have any real value. But we shall consider only rational values.

- $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{7}$, $\sqrt[5]{8}$, are some radicals. $\sqrt{0}$, $\sqrt[3]{0}$ are also radicals. $\sqrt[0]{0}$ and $\sqrt[-1]{0}$ are undefined. $\sqrt[3]{-2}$, $\sqrt[5]{-64}$, $\sqrt[7]{-128}$ are also radicals. $\sqrt[2]{-4}$, $\sqrt[4]{-16}$ are also radicals, but because they are not real numbers, we shall study them only in higher classes. In all these examples, the value of n, i.e., the order of the radical is a positive integer. But as stated above, it can have other values.

2.2 Description of surds

If a is a positive rational number, which is not the nth power (n is any natural number) of any rational number, then the irrational number $\pm\sqrt[n]{a}$ are called simple surds or monomial surds. Every surd is an irrational number (but every irrational number is not a surd). So, the representation of monomial surd on a number line is same that of irrational numbers. Eg.

(a) $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number.

(b) $\sqrt[3]{5}$ is surd and $\sqrt[3]{5}$ is an irrational number.

(c) π is an irrational number, but it is not a surd.

(d) $\sqrt[3]{3+\sqrt{2}}$ is an irrational number. It is not a surd, because $3+\sqrt{2}$ is not a rational number.

NUMERICAL CHALLENGE 2.1

Which of the following are surds?

(i) $\sqrt{9}$ (ii) $\sqrt[3]{13}$ (iii) $\sqrt[4]{25}$ (iv) $\sqrt[6]{32}$

Solution

(i) $\sqrt{9} = 3$ is not a surd.

(ii) $\sqrt[3]{13}$ is a surd.

(iii) $\sqrt[4]{25}$ is a surd.

(iv) $\sqrt[6]{32}$ is a surd.

2.3 Types of surds

Unit surds and multiples of surds

If $\sqrt[n]{a}$ is a surd, it is also referred to as a unit surd. If k is a rational number, $k\sqrt[n]{a}$ is a multiple of a surd.

- All multiples of surds can be expressed as unit surds as $k\sqrt[n]{a} = \sqrt[n]{k^n a}$.

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Mixed surds

If a is a rational number (not equal to 0) and $\sqrt[n]{b}$ is a surd, then $a + \sqrt[n]{b}$, $a - \sqrt[n]{b}$ are called mixed surds.

If $a = 0$, they are called pure surds. Eg. $2 + \sqrt{3}$, $5 - \sqrt[3]{6}$ are mixed surds, while $\sqrt{3}$, $\sqrt[3]{6}$ are pure surds.

Compound surd

A surd which is the sum or difference of two or more surds is called a compound surd. E.g.

$\sqrt{2} + \sqrt[3]{3}$, $\sqrt{3} + \sqrt[5]{7} - \sqrt[3]{2}$ and $1 + \sqrt{2} - \sqrt{3}$ are compound surds.

Binomial surd

A compound surd consisting of two surds is called a binomial surd. E.g.

$\sqrt{3} + 3\sqrt{5}$, $\sqrt{6} + 4\sqrt{5}$, $\sqrt{8} - \sqrt[3]{7}$

Similar surds

If two surds are different multiples of the same surd, they are called similar surds. Otherwise they are dissimilar surds. E.g.

$2\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$ are similar surds. $(\sqrt{2} + 3\sqrt{3})$, $(2\sqrt{2} + 6\sqrt{3})$ are similar surds and $1 + \sqrt{2}$, $2 + 2\sqrt{2}$ are similar surds. $3\sqrt{3}$ and $6\sqrt{5}$ are dissimilar surds.

- Equality of two mixed surds of the form $a + c\sqrt{b}$ and $d + e\sqrt{b}$.

  Two mixed surds $a + c\sqrt{b}$ and $d + e\sqrt{b}$ are equal if and only if their respective rational parts and the irrational parts are equal, i.e., $a = d$ and $c = e$.

NUMERICAL CHALLENGE 2.2

Identify the following types of surds:

(i) $\sqrt{6} + 5\sqrt{3}$ (ii) $\sqrt{15} + \sqrt{8} + \sqrt{11}$ (iii) $\sqrt{5}$ (iv) $5 + \sqrt{7}$

Solution

(i) $\sqrt{6} + 5\sqrt{3}$. It is the sum of two surds.

$\therefore$ It is a compound surd of two surds, i.e., a binomial surd.

(ii) $\sqrt{15} + \sqrt{8} + \sqrt{11}$. It is the sum of the three surds.

$\therefore$ It is a compound surd.

(iii) $\sqrt{5}$. It is a monomial surd or a simple surd.

(iv) $5 + \sqrt{7}$. It is the sum of a rational number and a surd.

$\therefore$ It is a mixed surd.

NUMERICAL CHALLENGE 2.3

Which of the following surds are similar?

(i) $2\sqrt{5}$ (ii) $3\sqrt[3]{5}$ (iii) $4\sqrt{5}$ (iv) $5\sqrt[4]{5}$

Solution

$2\sqrt{5}$ and $4\sqrt{5}$ are multiples of the same surd, $\sqrt{5}$.

$\therefore$ They are similar.

2.4 Laws of radicals

If $a > 0$, $b > 0$ and n is a positive rational number, then

(a) $(\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$

(b) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

(c) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

Laws of radicals are used to

(i) convert the multiple of a surd into a unit surd.

(ii) convert certain unit surds into multiples of other unit surds.

(iii) express surds of different orders as surds of the same order.

NUMERICAL CHALLENGE 2.4

Express the following surds in their simplest form as multiples of smaller surds:

(i) $\sqrt[3]{1458}$ (ii) $\sqrt[3]{144}$ (iii) $\sqrt[4]{1024}$

Solution

(i) $\sqrt[3]{1458} = \sqrt[3]{2(9^3)} = 9\sqrt[3]{2}$

(ii) $\sqrt[3]{144} = \sqrt[3]{2^4(3^2)} = \sqrt[3]{2^3(2)(3^2)} = 2\sqrt[3]{18}$

(iii) $\sqrt[4]{1024} = \sqrt[4]{2^{10}} = \sqrt[4]{(2^8)(2^2)} = \sqrt[4]{2^8}\sqrt[4]{2^2} = 4\sqrt{2}$

Order of surd

In the surd $\sqrt[n]{a}$, n is called the order of the surd. Thus, the order of $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$ are 2, 3 and 4 respectively.

- The orders of the radicals $\sqrt[4]{9}$, $\sqrt[6]{16}$, $\sqrt[6]{27}$ are 4, 6 and 6 respectively. We note that $\sqrt[4]{9} = \sqrt[4]{3^2} = 3^{2/4} = \sqrt{3}$ is 2. Thus, the order of a surd is not a property of the surd itself, but of the way in which it is expressed.

Comparison of monomial surds

If two simple surds are of the same order, then they can easily be compared. If $a < b$, $\sqrt[n]{a} < \sqrt[n]{b}$ for all positive integral values of n. E.g. $\sqrt{2} < \sqrt{7}$, $\sqrt[3]{3} < \sqrt[3]{5}$, $\sqrt[5]{10} < \sqrt[5]{13}$ etc.

If two simple surds of different orders have to be compared, they have to be expressed as radicals of the same order.

Thus to compare $\sqrt[4]{6}$ and $\sqrt[3]{5}$, we express both as the radicals of 12 (LCM of 3, 4) order.

$\sqrt[4]{6} = \sqrt[12]{6^3}$ and $\sqrt[3]{5} = \sqrt[12]{5^4}$

As $6^3 < 5^4$, $\sqrt[4]{6} < \sqrt[3]{5}$

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NUMERICAL CHALLENGE 2.5

Arrange the following in ascending or descending order of magnitude.

$\sqrt[4]{6}$, $\sqrt[3]{7}$, $\sqrt{5}$

Solution

$\sqrt[4]{6} = 6^{1/4}$, $\sqrt[3]{7} = 7^{1/3}$, $\sqrt{5} = 5^{1/2}$, LCM of the denominators of the exponents of the three terms, 4, 3 and 2 is 12.

Now express the exponent of each term, as a fraction in which the denominator is 12.

$6^{1/4} = 6^{3/12} = (6^3)^{1/12} = \sqrt[12]{216}$

$7^{1/3} = 7^{4/12} = (7^4)^{1/12} = \sqrt[12]{2401}$

$5^{1/2} = 5^{6/12} = (5^6)^{1/12} = \sqrt[12]{15625}$

Now $\sqrt[4]{6} = \sqrt[12]{216}$, $\sqrt[3]{7} = \sqrt[12]{2401}$, $\sqrt{5} = \sqrt[12]{15625}$

Hence, their ascending order is

$\sqrt[12]{216}$, $\sqrt[12]{2401}$, $\sqrt[12]{15625}$, i.e., $\sqrt[4]{6}$, $\sqrt[3]{7}$, $\sqrt{5}$

The descending order of magnitude of the given radicals is $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[4]{6}$.

2.5 Operations of surds

Addition and subtraction of surds

Addition and subtraction of similar surds can be done using the distributive law, $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$.

NUMERICAL CHALLENGE 2.6

Simplify the following by combining similar surds.

(i) $2\sqrt{5} + 5\sqrt{5}$

(ii) $3\sqrt{6} + \sqrt{216}$

(iii) $2\sqrt{3} - 5\sqrt{12} + 3\sqrt{48}$

Solution

(i) $2\sqrt{5} + 5\sqrt{5} = (2+5)\sqrt{5} = 7\sqrt{5}$

(ii) $3\sqrt{6} + \sqrt{216} = 3\sqrt{6} + \sqrt{6^2(6)} = 3\sqrt{6} + 6\sqrt{6} = 9\sqrt{6}$

(iii) $2\sqrt{3} - 5\sqrt{12} + 3\sqrt{48}$

$= 2\sqrt{3} - 5\sqrt{2^2(3)} + 3\sqrt{4^2(3)}$

$= 2\sqrt{3} - 5(2)\sqrt{3} + 3(4)\sqrt{3}$

$= (2 - 10 + 12)\sqrt{3} = 4\sqrt{3}$

Multiplication and division of surds

Surds of the same order can be multiplied according to the law, $(\sqrt[n]{x})(\sqrt[n]{y}) = \sqrt[n]{xy}$

- When the surds to be multiplied or divided are not of the same order, they have to be necessarily brought to the same order before the operation is performed.

NUMERICAL CHALLENGE 2.7

Simplify the following

(i) $\sqrt{15} \times \sqrt{35}$

(ii) $2\sqrt{3} \div 3\sqrt{27}$

(iii) Multiply $\sqrt[3]{3}$ by $\sqrt[4]{2}$

(iv) Divide $\sqrt[6]{5}$ by $\sqrt[3]{10}$

Solution

(i) $(\sqrt{15})(\sqrt{35}) = \sqrt{(15)(35)} = \sqrt{(5)(3)(5)(7)} = 5\sqrt{21}$

(ii) $2\sqrt{3} \div 3\sqrt{27} = \dfrac{2\sqrt{3}}{3\sqrt{27}} = \dfrac{2\sqrt{3}}{(3)\sqrt{3^2(3)}} = \dfrac{2\sqrt{3}}{(3)(3)\sqrt{3}} = \dfrac{2}{9}$

(iii) $\sqrt[3]{3} = 3^{1/3}$ and $\sqrt[4]{2} = 2^{1/4}$

The LCM of 3 and 4 is 12

$\therefore 3^{1/3} = 3^{4/12} = \sqrt[12]{3^4}$, $2^{1/4} = 2^{3/12} = \sqrt[12]{2^3}$

$(\sqrt[3]{3})(\sqrt[4]{2}) = (\sqrt[12]{3^4})(\sqrt[12]{2^3}) = \sqrt[12]{(3^4)(2^3)} = \sqrt[12]{(81)(8)} = \sqrt[12]{648}$

(iv) $\sqrt[6]{5} = 5^{1/6}$ and $\sqrt[3]{10} = 10^{1/3}$

LCM of 3 and 6 is 6

$\sqrt[3]{10} = 10^{1/3} = 10^{2/6} = \sqrt[6]{10^2} = \sqrt[6]{100}$

$\therefore \dfrac{\sqrt[6]{5}}{\sqrt[3]{10}} = \dfrac{\sqrt[6]{5}}{\sqrt[6]{100}} = \sqrt[6]{\dfrac{5}{100}} = \sqrt[6]{\dfrac{1}{20}}$

2.6 Rationalizing factor (RF)

If the product of two surds is a rational number, then each of the two is a RF of the other.

- RF is not unique.
- If one RF of a surd is known, then the product of this factor and any non-zero rational number is also the RF of the given surd.
- It is convenient to use the simplest of all RF's of the given surd to convert it to a rational number.

E.g. (a) $(3\sqrt{3})(\sqrt{3}) = (3)(3) = 9$, a rational number

$\therefore \sqrt{3}$ is a RF of $3\sqrt{3}$

(b) $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$, a rational number

$\therefore \sqrt{3}-\sqrt{2}$ is a RF of $\sqrt{3}+\sqrt{2}$ and $\sqrt{3}+\sqrt{2}$ is a RF of $\sqrt{3}-\sqrt{2}$

NUMERICAL CHALLENGE 2.8

Find the simplest RF of

(i) $\sqrt[4]{216}$

(ii) $\sqrt[5]{16}$

Solution

(i) $\sqrt[4]{216} = \sqrt[4]{(2^3)(3^3)} = 2^{3/4} \times 3^{3/4}$

So RF $= 2^{1/4} \times 3^{1/4} = (2 \times 3)^{1/4} = 6^{1/4} = \sqrt[4]{6}$

$\therefore \sqrt[4]{6}$ is the simplest RF of $\sqrt[4]{216}$

(ii) $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{4/5}$

$\therefore$ RF is $2^{1/5} \Rightarrow (2^{4/5})(2^{1/5}) = 2^{5/5} = 1$

$\therefore \sqrt[5]{2}$ is the simplest RF of $\sqrt[5]{16}$

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- $\sqrt[n]{a}$ is a RF of $\sqrt[n]{a^{n-1}}$ and vice-versa.
- $\sqrt[n]{a^m}$ is a RF of $\sqrt[n]{a^{n-m}}$ and vice-versa.
- $\sqrt{a}+\sqrt{b}$ is a RF of $\sqrt{a}-\sqrt{b}$ and vice-versa.
- $\sqrt[3]{a}+\sqrt[3]{b}$ is a RF of $a^{2/3}-a^{1/3}b^{1/3}+b^{2/3}$ and vice-versa.
- $\sqrt[3]{a}-\sqrt[3]{b}$ is a RF of $a^{2/3}+a^{1/3}b^{1/3}+b^{2/3}$ and vice-versa.

NUMERICAL CHALLENGE 2.9

Express the following surds with rational denominators:

(i) $\frac{2}{\sqrt{14}}$ (ii) $\frac{2\sqrt[3]{3}}{\sqrt[3]{25}}$

Solution

(i) $\frac{2}{\sqrt{14}}\times\frac{\sqrt{14}}{\sqrt{14}}=\frac{2\sqrt{14}}{14}=\frac{\sqrt{14}}{7}$ (ii) $\frac{2\sqrt[3]{3}}{\sqrt[3]{25}}=\frac{2\sqrt[3]{3}}{\sqrt[3]{25}}\times\frac{\sqrt[3]{5}}{\sqrt[3]{5}}=\frac{2\sqrt[3]{3\times5}}{\sqrt[3]{5^3}}=\frac{2\sqrt[3]{15}}{5}$

NUMERICAL CHALLENGE 2.10

Given that $\sqrt{2}=1.414$, find the value of $\frac{3}{\sqrt{2}}$ upto three decimal places.

Solution

$\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2}=1.5\sqrt{2}=1.5(1.414)=2.121$

2.7 Rationalization of mixed surds

$(a+\sqrt{b})$ is the rationalizing factor of $a-\sqrt{b}$, where a and b are rational.

NUMERICAL CHALLENGE 2.11

Rationalize the denominator of $\frac{2+\sqrt{5}}{2-\sqrt{5}}$.

Solution

$\frac{2+\sqrt{5}}{2-\sqrt{5}}=\left[\frac{2+\sqrt{5}}{2-\sqrt{5}}\right]\left[\frac{2+\sqrt{5}}{2+\sqrt{5}}\right]$

$=\frac{(2+\sqrt{5})^2}{(2)^2-(\sqrt{5})^2}=\frac{4+4\sqrt{5}+5}{4-5}=\frac{9+4\sqrt{5}}{-1}=-(9+4\sqrt{5})$

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NUMERICAL CHALLENGE 2.12

Given $\sqrt{2}=1.414$, $\sqrt{3}=1.732$, $\sqrt{5}=2.236$, $\sqrt{6}=2.449$, $\sqrt{10}=3.162$

Find the value of $\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}$ upto three decimal places.

Solution

We have to rationalize the denominator

The RF of $\sqrt{3}-\sqrt{5}$ is $\sqrt{3}+\sqrt{5}$.

$\therefore\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}=\left[\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{5}}\right]\left[\frac{\sqrt{3}+\sqrt{5}}{\sqrt{3}+\sqrt{5}}\right]$

$=\frac{\sqrt{6}-\sqrt{3}+\sqrt{10}-\sqrt{5}}{(\sqrt{3})^2-(\sqrt{5})^2}=\frac{\sqrt{10}+\sqrt{6}-\sqrt{3}-\sqrt{5}}{3-5}=\frac{3.162+2.499-1.372-2.236}{-2}=-0.822$

Conjugate of the surd of the form $(\sqrt{a}+\sqrt{b})$

When two binomial quadratic surds differ only in sign which connect their terms, they are said to be conjugate.

Two binomial quadratic surds $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$, are called conjugate surds. The product of conjugate surds is rational.

NUMERICAL CHALLENGE 2.13

Write the conjugate of

(i) $\sqrt{3}+\sqrt{5}$ (ii) $5+3\sqrt{7}$

Solution

(i) $\sqrt{3}-\sqrt{5}$ is the conjugate of $\sqrt{3}+\sqrt{5}$ and $\sqrt{3}-\sqrt{5}$ is also the conjugate of $\sqrt{3}+\sqrt{5}$.

(ii) $5-3\sqrt{7}$ is the conjugate of $5+3\sqrt{7}$.

- The following formulae are helpful in finding the rationalizing factors of mixed quadratic and cubic surds.
  (i) $(a+b)(a-b)=a^2-b^2$
  (ii) $(a+b)(a^2-ab+b^2)=a^3+b^3$
  (iii) $(a-b)(a^2+ab+b^2)=a^3-b^3$

NUMERICAL CHALLENGE 2.14

Find the RF

(i) $2^{1/3}+2^{-1/3}$. (ii) $5^{1/3}-5^{-1/3}$.

Solution

(i) $2^{1/3}+2^{-1/3}$

Let $a=2^{1/3}$ and $b=2^{-1/3}$

$a^3=(2^{1/3})^3=2$, $b^3=(2^{-1/3})^3=2^{-1}=\frac{1}{2}$

But $a^3+b^3=(a+b)(a^2-ab+b^2)$

$\therefore a^2-ab+b^2=(2^{1/3})^2-(2^{1/3}.2^{-1/3})+(2^{-1/3})^2=2^{2/3}-1+2^{-2/3}$

$\therefore$ RF of $2^{1/3}+2^{-1/3}$ is $2^{2/3}-1+2^{-2/3}$

(ii) $5^{1/3}-5^{-1/3}$

$\therefore(a^3-b^3)=(a-b)(a^2+ab+b^2)$

$\therefore$ RF of $5^{1/3}-5^{-1/3}$ is $(5^{1/3})^2+(5^{1/3}.5^{-1/3})+(5^{-1/3})^2=5^{2/3}+1+5^{-2/3}$

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Comparison of compound surds

NUMERICAL CHALLENGE 2.15

Between $\sqrt{7}-\sqrt{3}$ and $\sqrt{11}-\sqrt{7}$, which is greater?

Solution

By rationalizing,

$$\sqrt{7}-\sqrt{3} = \frac{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})}{\sqrt{7}+\sqrt{3}} = \frac{4}{\sqrt{7}+\sqrt{3}}$$

$$\sqrt{11}-\sqrt{7} = \frac{(\sqrt{11}-\sqrt{7})(\sqrt{11}+\sqrt{7})}{\sqrt{11}+\sqrt{7}} = \frac{4}{\sqrt{11}+\sqrt{7}}$$

The numerator of each of the irrational number is 4.

But $\sqrt{11}+\sqrt{7} > \sqrt{7}+\sqrt{3}$

$$\therefore \frac{4}{\sqrt{7}+\sqrt{3}} > \frac{4}{\sqrt{11}+\sqrt{7}} \cdot \sqrt{7}-\sqrt{3} > \sqrt{11}-\sqrt{7}$$

NUMERICAL CHALLENGE 2.16

Compare the surds $A = \sqrt{8}+\sqrt{7}$ and $B = \sqrt{10}+\sqrt{5}$.

Solution

Since there is a positive sign, square both the surds, we get,

$$A^2 = \left(\sqrt{8}+\sqrt{7}\right)^2 = 8+7+2\sqrt{56} = 15+2\sqrt{56}$$

$$B^2 = \left(\sqrt{10}+\sqrt{5}\right)^2 = 10+5+2\sqrt{50} = 15+2\sqrt{50}$$

As $56 > 50$, $15+2\sqrt{56} > 15+2\sqrt{50} \Rightarrow A > B$

i.e., $\sqrt{8}+\sqrt{7} > \sqrt{10}+\sqrt{5}$

Rationalizing the numerator

NUMERICAL CHALLENGE 2.17

Rationalize the numerator of $\frac{2-\sqrt{3+x}}{x-1}$.

Solution

Rationalizing factor of $2-\sqrt{3+x}$ is $2+\sqrt{3+x}$

$$\therefore \frac{2-\sqrt{3+x}}{x-1} = \left[\frac{2-\sqrt{3+x}}{x-1}\right]\left[\frac{2+\sqrt{3+x}}{2+\sqrt{3+x}}\right]$$

$$= \frac{(2)^2-(\sqrt{3+x})^2}{(x-1)(2+\sqrt{3+x})} = \frac{4-(3+x)}{(x-1)(2+\sqrt{3+x})} = \frac{1-x}{(x-1)(2+\sqrt{3+x})} = \frac{-1}{2+\sqrt{3+x}}$$

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NUMERICAL CHALLENGE 2.18

Express $E = \frac{1}{\sqrt{5}+\sqrt{3}-\sqrt{8}}$ with a rational denominator.

Solution

The denominator is a trinomial surd, an expression having all the three as surds. We group any two of the three terms, say $\sqrt{5}$ and $\sqrt{3}$.

Thus $\sqrt{5}+\sqrt{3}-\sqrt{8} = (\sqrt{5}+\sqrt{3})-\sqrt{8}$.

Consider the product

$$\left[(\sqrt{5}+\sqrt{3})-\sqrt{8}\right]\left[(\sqrt{5}+\sqrt{3})+\sqrt{8}\right] = \left(\sqrt{5}+\sqrt{3}\right)^2-\left(\sqrt{8}\right)^2$$

$$= 5+3+2\sqrt{5}\sqrt{3}-8 = 2\sqrt{15}$$

$$\therefore \frac{1}{\sqrt{5}+\sqrt{3}-\sqrt{8}} = \frac{\sqrt{5}+\sqrt{3}+\sqrt{8}}{(\sqrt{5}+\sqrt{3}-\sqrt{8})(\sqrt{5}+\sqrt{3}+\sqrt{8})} = \frac{(\sqrt{5}+\sqrt{3}+\sqrt{8})}{2\sqrt{15}}.$$

Rationalizing the denominator

$$E = \frac{\sqrt{5}+\sqrt{3}+\sqrt{8}}{2\sqrt{15}}\left(\frac{\sqrt{15}}{\sqrt{15}}\right) = \frac{5\sqrt{3}+3\sqrt{5}+2\sqrt{30}}{30}$$

NUMERICAL CHALLENGE 2.19

If both a and b are rational numbers, find the value of a and b in each of the following.

(i) $\frac{3+\sqrt{5}}{3-\sqrt{5}} = a+b\sqrt{5}$ (ii) $\frac{3+2\sqrt{3}}{5-2\sqrt{3}} = a+b\sqrt{3}$

Solution

(i) $3+\sqrt{5}$ is the rationalizing factor of $3-\sqrt{5}$.

$$\therefore \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{3+\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{\left(3+\sqrt{5}\right)^2}{(3)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{9+5+6\sqrt{5}}{9-5} = \frac{14+6\sqrt{5}}{4} = \frac{14}{4}+\frac{6}{4}\sqrt{5} = \frac{7}{2}+\frac{3}{2}\sqrt{5} = a+b\sqrt{5}$$

$$\therefore a = \frac{7}{2} \text{ and } b = \frac{3}{2}$$

(ii) $5+2\sqrt{3}$ is the RF of $5-2\sqrt{3}$.

$$\therefore \frac{3+2\sqrt{3}}{5-2\sqrt{3}} = \frac{3+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}} = \frac{15+10\sqrt{3}+12+6\sqrt{3}}{(5)^2-(2\sqrt{3})^2}$$

$$= \frac{27+16\sqrt{3}}{25-12} = \frac{27+16\sqrt{3}}{13} = a+b\sqrt{3} \Rightarrow a = \frac{27}{13} \text{ and } b = \frac{16}{13}.$$

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2.8 Square root of a quadric surd

Consider the real number $a + \sqrt{b}$, where a and b rational numbers and $\sqrt{b}$ is a surd. Equate the square root of $a + \sqrt{b}$ to $\sqrt{x} + \sqrt{y}$, where x and y are rational numbers, i.e

$\sqrt{a+\sqrt{b}} = \sqrt{x} + \sqrt{y}$

Squaring both sides, $a + \sqrt{b} = x + y + 2\sqrt{xy}$

Equating the rational numbers on the two sides of the above equation we get $a = x + y$ .....(1)

and equating the irrational numbers, we get

$\sqrt{b} = 2\sqrt{xy}$ .....(2)

By solving (1) and (2) we get the values of x and y.

Similarly, $\sqrt{a-\sqrt{b}} = \sqrt{x} - \sqrt{y}$

2.9 Square root of a trinomial quadratic surd

Consider the real number $a + \sqrt{b} + \sqrt{c} + \sqrt{d}$, where a is a rational number and $\sqrt{b}$, $\sqrt{c}$ and $\sqrt{d}$ are surds.

$\sqrt{a+\sqrt{b}+\sqrt{c}+\sqrt{d}} = \sqrt{x} + \sqrt{y} + \sqrt{z}$

By squaring both sides, and comparing rational and irrational parts on either sides, we get,

$x + y + z = a$

$x = \frac{1}{2}\sqrt{\frac{bd}{c}}$, $y = \frac{1}{2}\sqrt{\frac{bc}{d}}$ and $z = \frac{1}{2}\sqrt{\frac{cd}{b}}$

NUMERICAL CHALLENGE 2.20

Find the square root of

(i) $7 + 4\sqrt{3}$ (ii) $10 + \sqrt{24} + \sqrt{60} + \sqrt{40}$

Solution

(i) Let $\sqrt{7+4\sqrt{3}} = \sqrt{x} + \sqrt{y}$

Squaring both the sides.

$7 + 4\sqrt{3} = x + y + 2\sqrt{xy} \Rightarrow x + y = 7$ and $2\sqrt{xy} = 4\sqrt{3} \Rightarrow xy = 12$

By solving, we get $x = 4$ and $y = 3$

$\sqrt{x} + \sqrt{y} = \sqrt{4} + \sqrt{3} = 2 + \sqrt{3}$

(ii) Let the given expression be equal to $a + \sqrt{x} + \sqrt{y} + \sqrt{z}$

As per the method discussed, $a = 10$, $b = 24$, $c = 60$ and $d = 40$

$x = \frac{1}{2}\sqrt{\frac{bd}{c}} = \frac{1}{2}\sqrt{\frac{24\times40}{60}} = 2$, $y = \frac{1}{2}\sqrt{\frac{bc}{d}} = \frac{1}{2}\sqrt{\frac{24\times60}{40}} = 3$

$z = \frac{1}{2}\sqrt{\frac{cd}{b}} = \frac{1}{2}\sqrt{\frac{60\times40}{24}} = 5$

$\sqrt{x} + \sqrt{y} + \sqrt{z} = \sqrt{2} + \sqrt{3} + \sqrt{5}$

Alternative method

$= \sqrt{10 + \sqrt{24} + \sqrt{60} + \sqrt{40}}$

$= \sqrt{10 + 2\sqrt{6} + 2\sqrt{15} + 2\sqrt{10}}$

$= \sqrt{(2+3+5) + 2\sqrt{2(3)} + 2\sqrt{3(5)} + 2\sqrt{2(5)}}$

$= \sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2}$

$= \sqrt{2} + \sqrt{3} + \sqrt{5}$

Other important results

- $\sqrt[3]{a+K\sqrt{b}} = \sqrt{\sqrt{\frac{4b^2+K}{5}} - b} + \sqrt{b}$
- $\sqrt[3]{a+b\sqrt{c}} = \sqrt{\frac{b-c}{3}} + \sqrt{c}$
- $\left(a+\sqrt{b}\right)^{x^2-K} + \left(a-\sqrt{b}\right)^{x^2-K} = 2a$,

  $a^2 - b = 1 \Rightarrow x = \pm\sqrt{K \pm 1}$

2.10 Indices

Some useful Formulae

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = (a^n)^m = a^{mn}$

(iv) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

(v) $a^m \div b^{-n} = a^m \times b^n$

(vi) $(\sqrt[n]{a})^n = a$, where 'n' is a positive integer and 'a' a positive rational number

(vii) $\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$, where 'n' is a positive integer and 'a', 'b' are rational numbers.

(viii) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$, where 'n' is a positive integer and 'a', 'b' are rational numbers.

(ix) $\sqrt[n]{\sqrt[m]{a}} = \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$, where 'm', 'n' are positive integers and 'a' is a positive rational number

(x) $\sqrt[n]{\sqrt[m]{(a^k)^m}} = \sqrt[n]{a^k} = \sqrt[m]{a^{km}}$, where 'm', 'n' are 'k' are positive integers and 'a' is a positive rational number

(xi) $\sqrt{a} \times \sqrt{a} = a$

(xii) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(xiii) $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$

(xiv) $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$

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(xv) $a+\sqrt{b}=c+\sqrt{d} \Rightarrow a=c$ and $b=d$.

(xvi) $\frac{1}{\sqrt{a}-\sqrt{b}}=\frac{\sqrt{a}+\sqrt{b}}{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}=\frac{\sqrt{a}+\sqrt{b}}{a-b}$

(xvii) $\frac{1}{\sqrt{a}+\sqrt{b}}=\frac{\sqrt{a}-\sqrt{b}}{(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})}=\frac{\sqrt{a}-\sqrt{b}}{a-b}$

(xviii) If $x=n(n+1)$, then

(a) $\sqrt{x-\sqrt{x-\sqrt{x-.....\infty}}}=n$

(b) $\sqrt{x+\sqrt{x+\sqrt{x+.....\infty}}}=(n+1)$

NUMERICAL CHALLENGE 2.21

Find the value of x

(i) $x^{x\sqrt{x}}=\left(x\sqrt{x}\right)^x$ (ii) $2^{x-2}=5^{2-x}$ (iii) $\frac{2^{x-1}.4^{2x+1}}{8^{x-1}}=64$

(iv) $2^{x+1}.5^x=200$ (v) $\frac{\left(\frac{1}{9}\right)^{2x-1}.(.0081)^{1/3}}{\sqrt{243}}=\left(\frac{1}{3}\right)^{2x-5}\sqrt[3]{\frac{27^{x-1}}{10000}}$

Solution

(i) $x^{x\sqrt{x}}=\left(x^{\frac{3}{2}}\right)^x \Rightarrow x^{x\sqrt{x}}=x^{\frac{3}{2}x}$

$x\sqrt{x}=\frac{3}{2}x \Rightarrow \sqrt{x}=\frac{3}{2} \Rightarrow x=\frac{9}{4}$

(ii) $2^{x-2}=5^{2-x} \Rightarrow 2^{x-2}=\frac{1}{5^{x-2}}$

$\Rightarrow 10^{x-2}=1 \Rightarrow \frac{10^x}{10^2}=1$

$\Rightarrow 10^x=10^2 \Rightarrow x=2.$

(iii) $\frac{2^{x-1}.4^{2x+1}}{8^{x-1}}=64$

$\Rightarrow \frac{2^{x-1}.2^{4x+2}}{2^{3x-3}}=2^6$ (Base has been made equal to 2)

$\Rightarrow 2^{x-1+4x+2-3x+3}=2^6$

$\Rightarrow 2^{x-1+4x+2-3x+3}=2^6$

$\Rightarrow 2^{2x+4}=2^6$

$\Rightarrow 2x+4=6$

$\Rightarrow x=1$

(iv) $2^{x+1}.5^x=200 \Rightarrow 2.2^x.5^x=200$

$\Rightarrow (2.5)^x=100 \Rightarrow 10^x=10^2$

$\Rightarrow x=2$

(v) The given equation is

$$\frac{(3^{-2})^{2x-1}.(3^4.10^{-4})^{1/3}}{3^{5/2}}=\frac{3^{-(2x-5)}.3^{3\left(\frac{x-1}{3}\right)}}{10^{4/3}}$$

$$\Rightarrow \frac{3^{-4x+2+4/3-5/2}}{10^{4/3}}=\frac{3^{-2x+5+x-1}}{10^{4/3}}$$

$\Rightarrow 3^{-4x+5/6}=3^{-x+4} \Rightarrow -4x+5/6=-x+4 \Rightarrow x=\frac{-19}{18}$

NUMERICAL CHALLENGE 2.22

If $3^{x+y}=81$ and $81^{x-y}=3$, then find the values of x and y.

Solution

We have, $3^{x+y}=81=3^4$ and $(3^4)^{x-y}=3^1$

$\Rightarrow x+y=4$ .....(1)

and $4x-4y=1$ .....(2)

on adding, we get $8x=17$

$\Rightarrow x=\frac{17}{8}$

Substituting the value of 'x' in (1), we get, $y=\frac{15}{8}$

NUMERICAL CHALLENGE 2.23

Prove that $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=1$

Solution

$$\text{LHS}\frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}+\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}=\frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^b+x^a+x^c}+\frac{x^c}{x^c+x^b+x^a}=1$$

= RHS.

NUMERICAL CHALLENGE 2.24

If $\frac{2^x}{1+2^x}=\frac{1}{4}$, then value of $\frac{8^x}{1+8^x}$.

Solution

$\frac{2^x}{1+2^x}=\frac{1}{4} \Rightarrow 4\cdot 2^x=1+2^x$

$\Rightarrow 3\cdot 2^x=1 \Rightarrow 2^x=\frac{1}{3}$

$\therefore \frac{8^x}{1+8^x}=\frac{(2^x)^3}{1+(2^x)^3}=\frac{\frac{1}{27}}{1+\frac{1}{27}}=\frac{1}{28}$

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NUMERICAL CHALLENGE 2.25

If $a^{1/x} = b^{1/y} = c^{1/z}$, $b^2 = ac$, then find the value of $\frac{x+z}{2y}$.

Solution

$a^{1/x} = k \quad b^{1/y} = k \quad c^{1/z} = k$

$a = k^x \quad b = k^y \quad c = k^z$

we have, $b^2 = ac$

$(k^y)^2 = k^x \times k^z$

$k^{2y} = k^x \times k^z$

$\Rightarrow 2y = x + z$

$\Rightarrow \frac{x+z}{2y} = 1$

NUMERICAL CHALLENGE 2.26

Simplify :

(i) $\left(\frac{x-y}{z}+\frac{y-z}{x}+\frac{z-x}{y}\right)\left(\frac{z}{x-y}+\frac{x}{y-z}+\frac{y}{z-x}\right)$ if $x + y + z = 0$

(ii) $\frac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6} = (x \neq 0, y \neq 0, z \neq 0)$

(iii) $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^{a-b}}\right)^{\frac{a^2}{b}}$

(iv) $\left(x^{2^{n-1}}+y^{2^{n-1}}\right)\left(x^{2^{n-1}}-y^{2^{n-1}}\right)$

Solution

(i) $\left(\frac{x-y}{z}+\frac{y-z}{x}+\frac{z-x}{y}\right)\left(\frac{z}{x-y}+\frac{x}{y-z}+\frac{y}{z-x}\right)$

$= 3+\sum_{x,y,z}\frac{(x-z)(x+z-y)y}{zx(z-x)}$

$= 3+\sum_{x,y,z}\frac{2y^2}{zx}$

$= 3 + 2\left(\frac{x^3+y^3+z^3}{xyz}\right)$

$= 3 + 6 = 9$

$= 3+\sum_{x,y,z}\left(\frac{x-y}{z}+\frac{y-z}{x}\right)\left(\frac{y}{z-x}\right)$

$= 3+\sum_{x,y,z}\frac{-(-2y)y}{zx}$ (since, $x + z = -y$)

$= 3 + 2\left(\frac{y^2}{zx}+\frac{z^2}{xy}+\frac{x^2}{xy}\right)$

$= 3 + 2\left(\frac{3xyz}{xyz}\right)$

(since, if $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$)

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(ii) $\frac{(xyz)^4}{(x^{-2}y^3)^{-3}\left(\frac{1}{z^2}\right)^6} = \frac{x^4.y^4.z^4}{x^6.y^{-9}.z^3} = \frac{y^{9+4}.z^{4-3}}{x^{6-4}} = \frac{y^{13}.z}{x^2}$

(iii) $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^{a-b}}\right)^{\frac{a^2}{b}} = x^{(a-b)(a+b)} \div (x^{a-a+b})^{\frac{a^2}{b}}$

$= x^{a^2-b^2} \div (x^b)^{\frac{a^2}{b}} = x^{a^2-b^2} \div x^{a^2} = x^{(a^2-b^2-a^2)} = x^{-b^2}$

(iv) In the given expression, we assume that

$x^{2^{n-1}} = a$, and $y^{2^{n-1}} = b$

Then it becomes

$(a + b)(a - b) = a^2 - b^2$

$= \left[x^{2^{n-1}}\right]^2 - \left[y^{2^{n-1}}\right]^2$

$= x^{2^{n-1}\times 2^1} = a$, and $y^{2^{n-1}\times 2^1}$

$= x^{2^{n-1+1}} - y^{2^{n-1+1}} = x^{2^n} - y^{2^n}$

NUMERICAL CHALLENGE 2.27

If $a^x = m$, $a^y = n$ and $a^2 = (m^y n^x)^z$, then find the value of xyz.

Solution

Given that $a^2 = (m^y n^x)^z$

$\Rightarrow a^2 = [(a^x)^y . (a^y)^x]^z \quad [\because m = a^x \; n = a^y]$

$\Rightarrow a^2 = [(a^{xy} . a^{xy}]^z$

$\Rightarrow a^2 = [(a^{2xy})^z$

$\Rightarrow a^2 = a^{2xyz}$

Since base is same.

Hence $2 = 2xyx$

$\Rightarrow xyz = 1$

NUMERICAL CHALLENGE 2.28

Which is the greater of the two 2^{300} or 3^{200}?

Solution

For comparing two numbers of different base and different exponents, it is better that either the base or the exponent is made same for both numbers.

Hence $2^{300} = (2^3)^{100} = 8^{100}$ (exponent is 100)

and $3^{200} = (3^2)^{100} = 9^{100}$ (exponent is 100)

From the above it is evident that

$9^{100} > 8^{100}$

$\Rightarrow 3^{200} > 2^{300}$

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NUMERICAL CHALLENGE 2.29

Given that $\sqrt[3]{3^x} = 5^{1/4}$ and $\sqrt[4]{5^y} = \sqrt{3}$, then find the value of 2xy.

Solution

$\sqrt[3]{3^x} = 5^{1/4}$

$\Rightarrow 3^{x/3} = 5^{1/4}$ ...(i)

and $\sqrt[4]{5^y} = \sqrt{3}$

$\Rightarrow 5^{y/4} = 3^{1/2}$

$\Rightarrow 5 = 3^{\frac{1}{2}\times\frac{4}{y}}$ ...(ii)

Putting the value of 5 from equation (ii) in equation (i).

$3^{x/3} = \left(3^{\frac{1}{2}\times\frac{4}{y}}\right)^{1/4}$

$\Rightarrow \frac{x}{3} = \frac{1}{2}\times\frac{4}{y}\times\frac{1}{4}$

$\Rightarrow 2xy = 3$

EXERCISE

Multiple choice questions

1. If $\sqrt{19-4\sqrt{x}} = \sqrt{12}-\sqrt{7}$, then x =
(1) 84 (2) 28 (3) 21 (4) 14

2. The mean proportional between $\sqrt{11}-\sqrt{5}$ and $13\sqrt{11}+19\sqrt{5}$ is: (geometric mean)
(1) $\sqrt{3}(\sqrt{11}-\sqrt{5})$ (2) $\sqrt{3}(\sqrt{11}+\sqrt{5})$ (3) $\sqrt{11}+\sqrt{5}$ (4) $\frac{\sqrt{11}+\sqrt{5}}{\sqrt{3}}$

3. $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+.....\infty}}}}$ =
(1) 3 (2) 2 (3) 1 (4) ±3

4. If $a = \sqrt{14}+\sqrt{18}$, $b = \sqrt{15}+\sqrt{17}$, then:
(1) a > b (2) a < b (3) a = b (4) a = 1.5b

5. If $x = 8-\sqrt{60}$, then $\frac{1}{2}\left[\sqrt{x}+\frac{2}{\sqrt{x}}\right]$ =
(1) $\sqrt{5}$ (2) $\sqrt{3}$ (3) $2\sqrt{5}$ (4) $2\sqrt{3}$

6. $\sqrt{\sqrt{50}+\sqrt{48}} = k(\sqrt{3}+\sqrt{2})$, then k =
(1) $2^{1/2}$ (2) 2 (3) $2^{1/4}$ (4) None

7. If $\frac{4+3\sqrt{5}}{4-3\sqrt{5}} = a+b\sqrt{5}$, then (a, b) =
(1) $\left(\frac{61}{29},\frac{-24}{29}\right)$ (2) $\left(\frac{-61}{29},\frac{24}{29}\right)$ (3) $\left(\frac{61}{29},\frac{24}{29}\right)$ (4) $\left(\frac{-61}{29},\frac{-24}{29}\right)$

8. If $x = \sqrt{5}$, $y = \sqrt[4]{10}$, $z = \sqrt[6]{6}$, then:
(1) x < y < z (2) y < z < x (3) z < x < y (4) None

9. If a, b, c are rational such that $a + b\sqrt{2} + c\sqrt{3} = 0$, then:
(1) b = c = 0 (2) a = 0 (3) a = b = c = 0 (4) None

10. If $x = \sqrt[3]{9}$, $y = \sqrt[4]{11}$, $z = \sqrt[6]{17}$ then:
(1) x > y > z (2) y > z > x (3) z > y > x (4) x < z < y

11. The smallest of $\sqrt[3]{4}$, $\sqrt[4]{5}$, $\sqrt[4]{6}$, $\sqrt[3]{8}$ is:
(1) $\sqrt[3]{8}$ (2) $\sqrt[4]{5}$ (3) $\sqrt[3]{4}$ (4) $\sqrt[4]{6}$

12. $(\sqrt{11}-\sqrt{10})\square(\sqrt{12}-\sqrt{11})$
(1) < (2) > (3) = (4) cannot be determined

13. If $x = \sqrt{2}+1$, $y = \sqrt{17}-\sqrt{2}$, then
(1) x < y (2) x > y (3) x = y (4) None

14. If $\frac{\ell}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}} = \frac{\sqrt{10}-\sqrt{14}-\sqrt{15}+\sqrt{21}}{k}$, then:
(1) $k = \ell/2$ (2) $\ell = k/2$ (3) $\ell = 2/k$ (4) None

15. The positive square root of $11+\sqrt{112}$ is:
(1) $\sqrt{7}+2$ (2) $\sqrt{7}+\sqrt{2}$ (3) $2-\sqrt{7}$ (4) None

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16. If $x = \sqrt{2} + 2^{1/3} + 2^{2/3}$, then $x^3 - 6x^2 + 6x =$
(1) 2 (2) 1 (3) 4 (4) 3

17. The square root $5 + 2\sqrt{6}$ is:
(1) $\sqrt{3}+2$ (2) $\sqrt{3}-\sqrt{2}$ (3) $\sqrt{2}-\sqrt{3}$ (4) $\sqrt{3}+\sqrt{2}$

18. If $(4+\sqrt{15})^{3/2} - (4-\sqrt{15})^{3/2} = k\sqrt{6}$, then k =
(1) 9 (2) 8 (3) 7 (4) 6

19. $\sqrt{21-4\sqrt{5}+8\sqrt{3}-4\sqrt{15}}$ =
(1) $\sqrt{5}-2+2\sqrt{3}$ (2) $\sqrt{5}-\sqrt{4}-\sqrt{12}$ (3) $-\sqrt{5}+\sqrt{4}+\sqrt{12}$ (4) $-\sqrt{5}-\sqrt{4}+\sqrt{12}$

20. $\frac{1}{\sqrt{12+\sqrt{140}}} + \frac{1}{\sqrt{8-\sqrt{60}}}$ =
(1) $\frac{2}{\sqrt{10+\sqrt{84}}}$ (2) 0 (3) $\frac{1}{\sqrt{10+\sqrt{84}}}$ (4) $\frac{2}{\sqrt{10-\sqrt{84}}}$

21. If $x = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$, then $x^2(x-14)^2$ =
(1) 1 (2) –1 (3) 2 (4) –2

22. If $\sqrt{23+x\sqrt{10}} = \sqrt{18}+\sqrt{5}$ then x =
(1) 6 (2) –6 (3) 2 (4) –2

23. If $\frac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = a + \sqrt{b}$, then (a, b) =
(1) (12, –1) (2) (1, 12) (3) (–1, 12) (4) (–12, 1)

24. If $x = a + \sqrt{a^2+1}$ then a in terms of x is
(1) $\frac{x^2-1}{2x}$ (2) $\frac{x^2+1}{2x}$ (3) $\frac{x^2-1}{2}$ (4) $\frac{x^2-3}{2}$

25. If $x = 3-\sqrt{5}$, then $\frac{\sqrt{x}}{\sqrt{2+\sqrt{3x-2}}}$ =
(1) $1/\sqrt{5}$ (2) $\sqrt{5}$ (3) $\sqrt{3}$ (4) $1/\sqrt{3}$

26. Rationalise the denominator of $\frac{a+\sqrt{a^2-1}}{a-\sqrt{a^2-1}}$ is
(1) $2a^2+2a\sqrt{a^2-1}-1$ (2) $2a\sqrt{a^2-1}-1$ (3) $\sqrt{a^2-1}-1$ (4) $2a-1$

27. $\sqrt{2x^2-1+2x\sqrt{x^2-1}}$ =
(1) $x-\sqrt{x^2-1}$ (2) $x+\sqrt{x^2-1}$ (3) $x+\sqrt{x^2+1}$ (4) $x-\sqrt{x^2+1}$

28. If $1 \le a \le 2$, then $\sqrt{a-2\sqrt{a-1}} - \sqrt{a+2\sqrt{a-1}}$ =
(1) 2 (2) $2\sqrt{a-1}$ (3) –2 (4) None

29. $\frac{1}{2-\sqrt{3}} - \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{4}{3-\sqrt{5}}$ =
(1) $3+\sqrt{2}+\sqrt{7}$ (2) $5+\sqrt{2}+\sqrt{5}$ (3) $\sqrt{2}+\sqrt{5}$ (4) $1-\sqrt{5}$

30. $\frac{1}{\sqrt{5}-\sqrt{5-\sqrt{24}}} + \frac{1}{\sqrt{5}-\sqrt{5+\sqrt{24}}}$ =
(1) 1/2 (2) $1/\sqrt{2}$ (3) 1/3 (4) $-1/\sqrt{3}$

31. If $\frac{4}{2+\sqrt{3}+\sqrt{7}} = \sqrt{a}+\sqrt{b}-\sqrt{c}$, then:
(1) a = 1, b = 4/3, c = 7/3 (2) a = 1, b = 2/3, c = 7/9
(3) a = 2/3, b = 1, c = 7/3 (4) a = 7/9, b = 4/3, c = 1

32. A rational number between $\frac{5}{6}$ and $\frac{6}{7}$ is
(1) $\frac{7}{84}$ (2) $\frac{1}{84}$ (3) $\frac{81}{74}$ (4) $\frac{71}{84}$

33. If $x = 5+2\sqrt{6}$, then $\sqrt{\frac{x}{2}} - \frac{1}{\sqrt{2x}}$ =
(1) 1 (2) 2 (3) 3 (4) 4

34. $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}} + \frac{2\sqrt{3}}{\sqrt{6}+2}$ =
(1) 2 (2) 1 (3) 0 (4) –1

35. $\frac{3+\sqrt{6}}{\sqrt{75}-\sqrt{48}-\sqrt{32}+\sqrt{50}}$ =
(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) $\sqrt{3}+\sqrt{2}$ (4) $\sqrt{3}-\sqrt{2}$

36. If $\sqrt{\frac{6+2\sqrt{3}}{33-19\sqrt{3}}} = a+b\sqrt{3}$, then a + b =
(1) 6 (2) 8 (3) 10 (4) 12

37. If $(5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10$, then x =
(1) 2, –2 (2) $\sqrt{2}$, $-\sqrt{2}$ (3) 2, $+\sqrt{2}$ (4) 2, –2, $\sqrt{2}$, $-\sqrt{2}$

38. If $x = 2+\sqrt{3}$, $xy = 1$, then $\frac{x}{\sqrt{2}+\sqrt{x}} + \frac{y}{\sqrt{2}-\sqrt{y}}$ =
(1) $\sqrt{2}$ (2) $\sqrt{3}$ (3) 1 (4) None

39. If $\frac{\left(p+\frac{1}{q}\right)^p\left(p-\frac{1}{q}\right)^q}{\left(q+\frac{1}{p}\right)^p\left(q-\frac{1}{p}\right)^q} = \left(\frac{p}{q}\right)^x$ then x =
(1) p – q (2) p + q (3) q – p (4) pq

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40. $\sqrt{\sqrt{3}-\sqrt{4+\sqrt{5}+\sqrt{17-4\sqrt{15}}}}$ =
(1) 1 (2) $\sqrt{-1}$ (3) –1 (4) $\sqrt{5}$

41. $\frac{5+\sqrt{3}}{2-\sqrt{3}}$ =
(1) $3+7\sqrt{3}$ (2) $13+\sqrt{3}$ (3) $1+\sqrt{3}$ (4) $13+7\sqrt{3}$

42. If $\sqrt{10+2\sqrt{21}} = \sqrt{7}+\sqrt{x}$ then x =
(1) 4 (2) 3 (3) 1 (4) 0

43. $x = \sqrt{\frac{7+4\sqrt{3}}{7-4\sqrt{3}}}$ then $x+\frac{1}{x}$ =
(1) 0 (2) –1 (3) 14 (4) 2

44. If $3+\sqrt{27}+\sqrt{75} = a+b\sqrt{3}$ then (a, b) =
(1) 3, 4 (2) 3, 8 (3) 8, 1 (4) 4, 2

45. If $\sqrt[3]{6y-9} = 3$ then y =
(1) 1 (2) 2 (3) 6 (4) 0

46. If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$ then $a^2b + ab^2$ =
(1) 0 (2) 6 (3) –6 (4) 1

47. $x = 7+4\sqrt{3}$ and xy = 1 then $\frac{1}{x^2}+\frac{1}{y^2}$ =
(1) 194 (2) 19 (3) 192 (4) 14

48. If $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c} = 0$ the $(a+b+c)^3$ =
(1) abc (2) 3abc (3) 9ac (4) 27abc

49. If a, b, c are real numbers then the value of $\sqrt{a^{-1}}.\sqrt{b^{-1}}.\sqrt{c^{-1}}.\sqrt{a}$ is
(1) 0 (2) 1 (3) $\frac{1}{ab}$ (4) ab

50. If $x = 5+2\sqrt{6}$ then $\frac{x-1}{\sqrt{x}}$ =
(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $\sqrt{3}$ (4) $2\sqrt{3}$

51. If $y = \sqrt[3]{3}+\frac{1}{\sqrt[3]{3}}$, then the value of $3y^3 - 9y$
(1) 0 (2) – 1 (3) 10 (4) All of these

52. The value of the expression: $\frac{(x^{a+b})^2 \times (x^{b+c})^2 \times (x^{c+a})^2}{(x^a x^b x^c)^4}$, if x = 2, a = 1, b = 2, c = 3 is
(1) 1 (2) 8 (3) 0 (4) None of these

53. If $27^k = \frac{9}{3^k}$, then the value of $\frac{1}{k^2}$ is
(1) 4 (2) 0 (3) Both (1) & (2) (4) None of these

54. If $4^x = 5^y = 20^z$, then the value of z is
(1) $\frac{2xy}{x+y}$ (2) – 2xy (3) 2xy (4) $\frac{xy}{x+y}$

55. If $x^{a^3}.x^{b^3}.x^{3ab(a+b)} = (2^5)^{25}$, and a + b = 5, then the value of x
(1) 2 (2) 0 (3) – 2 (4) – 1

56. If $3^{2x^2} - 2.3^{x^2+x+6} + 3^{2(x+6)} = 0$ then the value of x is
(1) – 2 (2) 3 (3) Both (1) and (2) (4) None of these

57. If $x^by = 2x - 3y^2$, then the value of $\left(\frac{1}{2}\right)^b.\frac{1}{\sqrt{3}}$ is
(1) 1 (2) 0 (3) – 1 (4) – 2

58. The numerical value of $\left(\frac{x^m}{x^n}\right)^{m+n-l} \times \left(\frac{x^n}{x^l}\right)^{n+l-m} \times \left(\frac{x^l}{x^m}\right)^{l+m-n}$ is
(1) 1 (2) 8 (3) 0 (4) None of these

59. The numerical value of $\left(x^{1/a-b}\right)^{1/c-a} \times \left(x^{1/b-c}\right)^{1/a-b} \times \left(x^{1/c-a}\right)^{1/b-c}$ is
(1) 1 (2) 8 (3) 0 (4) None of these

60. The value of $\frac{\left(x^{2n-1}+y^{2n-1}\right)\left(x^{2n-1}-y^{2n-1}\right)}{x^{2n}-y^{2n}}$ is
(1) 1 (2) 8 (3) 0 (4) None of these

61. If $64^x = \frac{1}{256^y}$, then the value of 3x + 4y is
(1) 1 (2) 8 (3) 0 (4) None of these

62. If $x = y^z$, $y = z^x$, $z = x^y$, then the value of x y z is
(1) 0 (2) –1 (3) 2 (4) 1

63. $\left(\left((625)^{-1/2}\right)^{-1/4}\right)^2$ =
(1) 4 (2) 5 (3) 2 (4) 3

64. $\left(5\left(8^{1/3}+27^{1/3}\right)^3\right)^{1/4}$ =
(1) 3 (2) 6 (3) 5 (4) 4

65. $(1^3 + 2^3 + 3^3 + 4^3)^{-3/2}$ =
(1) 10^{-3} (2) 10^{-2} (3) 10^{-4} (4) 10^{-1}

66. $(0.000729)^{-3/4} \times (0.09)^{-3/4}$ =
(1) $\frac{10^3}{3^3}$ (2) $\frac{10^5}{3^5}$ (3) $\frac{10^2}{3^2}$ (4) $\frac{10^6}{3^6}$

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67. $\left\{\sqrt[4]{\left(\frac{1}{x}\right)^{-12}}\right\}^{-2/3}$ =

(1) $\frac{1}{x^2}$ (2) $\frac{1}{x^4}$ (3) $\frac{1}{x^3}$ (4) $\frac{1}{x}$

68. $\frac{\sqrt{x^3}\times\sqrt[3]{x^5}}{\sqrt[5]{x^3}}\times\sqrt[30]{x^{77}}$ =

(1) $x^{76/15}$ (2) $x^{78/15}$ (3) $x^{79/15}$ (4) $x^{77/15}$

69. If $800 = 8\times10^8\times x^{-3/2}$, then x=

(1) 10^2 (2) 10^3 (3) 10^4 (4) 10^5

70. $\left(-\frac{5}{6}\right)^{3/4}$ when divided by $\left(-\frac{5}{6}\right)^{7/6}$ becomes $\left(-\frac{5}{6}\right)^{7-x}$, the value of x is

(1) $\frac{7}{12}$ (2) $\frac{89}{12}$ (3) $\frac{8}{12}$ (4) $\frac{10}{12}$

71. If $4^x - 4^{x-1} = 24$, then the value of $(2x)^x$ is

(1) $5^{5/2}$ (2) $5^{1/3}$ (3) $4^{1/2}$ (4) $6^{1/2}$

72. If $a = b^{2x}$, $b = c^{2y}$, $c = a^{2z}$, then xyz =

(1) 2^{-2} (2) 2^{-3} (3) 2^2 (4) 2^3

73. If $\sqrt[4]{\sqrt[3]{x^2}} = x^k$, then k =

(1) $\frac{2}{6}$ (2) 6 (3) $\frac{1}{6}$ (4) 7

74. Given that $m^2 = 27^{2/3}\times16^{-3/2}$, find the value of m.

(1) $\frac{9}{34}$ (2) $\frac{3}{16}$ (3) $\frac{3}{8}$ (4) $\frac{3}{4}$

75. $\left(7^{-\frac{1}{2}}\times5^2\right)^2+\sqrt{25^3}$ =

(1) $\frac{5}{7}$ (2) $\frac{7}{5}$ (3) 35 (4) $-\frac{5}{7}$

76. $(2d^2e^{-1})^3\times\left(\frac{d^3}{e}\right)^{-2}$ =

(1) $8e^{-2}$ (2) $8e^{-3}$ (3) $8e^{-1}$ (4) $8e^{-4}$

77. $(27^{-2})^{1/3}\times\sqrt{4^3}$ =

(1) 7/9 (2) 8/9 (3) 9/9 (4) 10/9

78. If $9^{1/2}\times3^m = (27)^m$, then the value of m is

(1) 1/4 (2) 1/5 (3) 1/3 (4) 1/2

79. $\frac{x-y}{\sqrt{x}+\sqrt{y}}$ =

(1) $\sqrt{x-y}$ (2) $\sqrt{x}+\sqrt{y}$ (3) $-\left(\sqrt{x}+\sqrt{y}\right)$ (4) $\sqrt{x}-\sqrt{y}$

80. If $\sqrt{9^x} = \sqrt[3]{9^2}$, then x =

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{1}{3}$ (4) $\frac{5}{3}$

81. $n\left[1+\frac{1}{n}\right]\left[1+\frac{1}{n+1}\right]\cdots\left[1+\frac{1}{n+m}\right]$ =

(1) n + m (2) n + m + 1 (3) m (4) None of these

82. If $a^{xy} = \left(a^x\right)^y$, then $x^{y-1}\cdot y^{x-1}$ =

(1) y^x (2) x^y (3) xy (4) $\frac{y}{x}$

83. If $a = x + \frac{1}{x}$, then $x^3 + x^{-3}$ =

(1) $a^3 + 3a$ (2) $a^3 - 3a$ (3) $a^3 + 3$ (4) $a^3 - 3$

84. If $a = x^{1/3} - x^{-1/3}$, then $a^3 + 3a$ =

(1) $x - x^{-1}$ (2) $x^{-1} - x$ (3) $x + x^{-1}$ (4) $x^{-1} + x$

85. If $a + b + c = 0$, then $(a^3 + b^3 + c^3)^2$ =

(1) $3a^2b^2c^2$ (2) 9abc (3) $9a^2b^2c^2$ (4) 27abc

86. If $x^y = y^x$ and $x = 2y$, then the values of x and y are

(1) x = 4, y = 2 (2) x = 3, y = 2 (3) x = 1, y = 1 (4) None of these

87. If $a^x = b^y = c^z$, abc = 1, then the value of xy + yz + zx is

(1) 1 (2) 2 (3) 0 (4) None of these

88. If $a^{x-1} = bc$, $b^{y-1} = ac$, $c^{z-1} = ab$, then the value of xy + yz + zx − xyz is

(1) 1 (2) 2 (3) 0 (4) None of these

89. If $(a^m)^n = a^{mn}$, then express 'm' in the terms of n is

(1) $n^{\left(\frac{1}{n-1}\right)}$ (2) $n^{\left(\frac{1}{n+1}\right)}$ (3) $n^{\left(\frac{1}{n}\right)}$ (4) None of these

90. If $\left(\frac{q}{p}\right)^{1-2x} = \sqrt{\frac{p}{q}}$ then the value x is

(1) 3/4 (2) 4/3 (3) 1 (4) None of these

91. If $4^{x-1}\left(\frac{1}{2}\right)^{3-2x} = (0.125)^x$, then the value x is

(1) 5/7 (2) 2/8 (3) 2/7 (4) None of these

92. The value of $\left\{\frac{a^{q-p}}{\sqrt[q]{a^{pq-q^2}}}\times a^{2(p-q)}\right\}^n$ is

(1) 2 (2) p (3) 1 (4) None of these

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93. If $a + b + c = 0$, then $x^{a^2b^{-1}c^{-1}} \cdot x^{a^{-1}b^2c^{-1}} \cdot x^{a^{-1}b^{-1}c^2} =$

(1) $x^{a^2b^2c^2}$ (2) $x^{1/a^2b^2c^2}$ (3) $x^{1/2}$ (4) x^3

94. If $ab = 1$, then $\frac{1}{1+a^{-1}} + \frac{1}{1+b^{-1}} =$

(1) b (2) 1 (3) a + b (4) a

95. If $a^x = b^y = c^z = abc$, then $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} =$

(1) −1 (2) xyz (3) 1 (4) $\frac{1}{xyz}$

96. If $a^p = x$, $a^q = y$, $x^q y^p = a^{2/r}$ then pqr =

(1) 1 (2) −1 (3) $x^{-1/p}$ (4) x^p

97. $2^{3^{4^5}} =$

(1) 2^{60} (2) 2^{932} (3) $(2^3)^{20}$ (4) None of these

98. $\frac{1}{1+x^{a-b}} + \frac{1}{1+x^{b-a}} =$

(1) $\frac{x^{ab}}{x^a + x^b}$ (2) $\frac{x^{ab}}{x^{a-b}}$ (3) 1 (4) $\frac{x^{ab}}{x^{b-a}}$

99. $x^{\frac{a+b-c}{(b-c)(c-a)}} \cdot x^{\frac{b+c-a}{(c-a)(a-b)}} \cdot x^{\frac{c+a-b}{(c-b)(a-b)}} =$

(1) x^{a+b+c} (2) x (3) 1 (4) 0

100. $\frac{2.3^{n+1} + 7.3^{n-1}}{3^{n+2} - 2\left(\frac{1}{3}\right)^{1-n}} =$

(1) 1 (2) 3 (3) −1 (4) 0

101. If $\frac{(2^{n+1})^m (2^2)^n 2^n}{(2^{m+1})^n 2^{2m}} = 1$, then m =

(1) 0 (2) 1 (3) n (4) 2n

102. If abc = 1, then $\sum (1 + a + b^{-1})^{-1} =$

(1) 0 (2) 1 (3) −1 (4) a + b + c

103. If $x^y = y^x$, then $\left(\frac{x}{y}\right)^{x/y} =$

(1) 0 (2) 1 (3) $\frac{x}{y^{-1}}$ (4) $x^{\frac{x}{y}}$

104. The value of $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$ is

(1) 1 (2) 2 (3) 3 (4) All of these

105. The value of $\frac{3.2^n - 4.2^n}{2^n - 2^{n-1}}$ is

(1) − 1 (2) − 3 (3) − 4 (4) − 2

106. The value of $\sqrt[ab]{\frac{x^a}{x^b}} \cdot \sqrt[bc]{\frac{x^b}{x^c}} \cdot \sqrt[ca]{\frac{x^c}{x^a}}$ is

(1) 1 (2) 0 (3) 3 (4) All of these

107. The value of $\frac{p + p^2 + p^3 + p^4 + p^5 + p^6 + p^7}{p^{-3} + p^{-4} + p^{-5} + p^{-6} + p^{-7} + p^{-8} + p^{-9}}$ is

(1) p^{10} (2) 1 (3) 0 (4) p^3

108. If $(p + q)^{-1} (p^{-1} + q^{-1}) = p^a q^b$, then the value of a + b is

(1) −2 (2) − 1 (3) 0 (4) 3

109. The simplest value of : $[1 - 1\{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$ is

(1) x (2) x^2 (3) 1 (4) 0

110. The value of $\sqrt[\ell+m]{\frac{x^{\ell^2}}{x^{m^2}}} \times \sqrt[m+n]{\frac{x^{m^2}}{x^{n^2}}} \times \sqrt[n+\ell]{\frac{x^{n^2}}{x^{\ell^2}}}$ is

(1) 2 (2) 0 (3) −1 (4) 1

111. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} =$ _____

(1) −1 (2) 1 (3) 2 (4) −2

112. $\left(\frac{64}{125}\right)^{-2/3} \div \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left\{\frac{\left(\sqrt{25}\right)}{\sqrt[3]{64}}\right\}^0 =$ _____

(1) $\frac{9}{2}$ (2) $\frac{9}{4}$ (3) $\frac{8}{2}$ (4) $\frac{8}{4}$

113. If $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$, then x = _____

(1) −2 (2) 4 (3) −6 (4) −4

114. If $\frac{4^{n+1} \cdot 2^n - 8^n}{2^{2n}} = \frac{3}{8}$, then n = _____

(1) −1 (2) −2 (3) −3 (4) −4

115. $\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$ = _____

(1) 2 (2) 4 (3) 1 (4) 3

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116. $\frac{1}{1+a^{m-n}}+\frac{1}{1+a^{n-m}}=$

(1) 1 (2) 3 (3) 5 (4) 4

117. The simplest value of $\frac{\left(p^2-\frac{1}{q^2}\right)^p\left(p-\frac{1}{q}\right)^{q-p}}{\left(q^2-\frac{1}{p^2}\right)^q\left(q+\frac{1}{p}\right)^{p-q}}$ is

(1) 1 (2) $\left(\frac{p}{q}\right)^{p-q}$ (3) $\left(\frac{p}{q}\right)^{p+q}$ (4) -1

118. The value of $\frac{1}{1+x^{b-c}+x^{a-c}}+\frac{1}{1+x^{c-a}+x^{b-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}$ is

(1) 1 (2) 8 (3) 0 (4) None of these

119. The value of $\left(\frac{x^m}{x^n}\right)^{m^2+mn+n^2}\times\left(\frac{x^n}{x^l}\right)^{n^2+nl+l^2}\times\left(\frac{x^l}{x^m}\right)^{l^2+lm+m^2}$ is

(1) 1 (2) 8 (3) 0 (4) None of these

120. If $pqr = 1$, then the value of $\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$ is

(1) 1 (2) 8 (3) 0 (4) None of these

ANSWERS

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	2	1	2	1	3	4	2	3	1	2	2	1	3	1
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	1	4	1	3	4	1	1	3	1	1	1	2	3	2	4
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	1	4	2	3	2	2	4	1	2	2	4	2	3	2	3
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	1	4	2	2	3	1	1	4	1	3	2	1	1	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans.	3	4	2	3	1	4	1	4	3	2	1	2	3	3	1
Que.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans.	3	2	4	4	2	2	1	2	1	3	1	3	3	1	1
Que.	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
Ans.	1	3	4	2	3	1	4	3	3	1	4	2	3	1	4
Que.	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Ans.	1	1	1	1	4	2	2	4	3	3	1	3	1	1	1

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